Investigations into haptic space and haptic perception of shape for active touch

(Onderzoekingen naar de haptische ruimte en de haptische waarneming van vorm voor actieve tast)

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Chapter 1

General introduction and overview

Introduction

This thesis presents a number of psychophysical investigations into haptic space and haptic perception of shape. The topics covered range from haptic perception of curvature and distance to the shape of haptically straight lines. Haptic perception is understood to include the two subsystems of the cutaneous sense and kinesthesis or proprioception. The cutaneous sense refers to awareness of stimulation of the skin through contact with some external object, and kinesthesis refers to awareness of static and dynamic body posture. There are a few common threads running through the chapters of this thesis.

Exploratory movements: The significance of active touch

A number of researchers have noted that there are close ties between a haptic stimulus and the particular arm movements made by an observer. Based on qualitative observations of haptic activity of observers touching a stimulus, they noticed that the nature of hand movements critically depends on the stimulus property to be perceived (e.g., Davidson et al., 1974; Klatzky and Lederman, 1987). For example, when asked to judge the weight of an object, observers will typically lift the object with their hands a number of times. However, they will manipulate the same object quite differently when estimating the temperature or feeling the surface texture (whether the surface is rough or smooth). In the former case, observers typically rest their hand on the object passively and maximize contact area; in the latter case, the preferred way of touching is to quickly rub the fingers back and forth over the surface. Conversely, if the observer is constrained to quickly rubbing the fingertips across the surface, judging the object's temperature proves to be much more difficult. These observations conform easily to our own introspective experiences. However, why exactly it is easier to judge a certain stimulus property when touching the stimulus in a specific way is still unclear. Apparently, the observer prefers to touch a stimulus in a particular way because that way of touching is – in some sense – optimal.

Gibson (1962) offers a theoretical approach to the role of hand movements in haptic perception. The stimulation of the sensory receptors depends on the particular object that is being touched but also on the observer's motor activity. Therefore, Gibson suggests that under conditions of free exploration (active touch) observers will make those kinds of arm movements that enhance certain features of the sensory stimulations while reducing others. It is assumed that these arm movements reinforce the sensory inflow related to the particular stimulus property Therefore, out of all the possible arm movements that an observer can make, these particular arm movements are most suited for picking up relevant information. Thus, when presented a stimulus, an observer will start exploring the object and search for the appropriate stimulation: An effort is made to obtain that kind of stimulation that yields perception of what is being touched. For that reason, Gibson calls these arm movements exploratory.

Despite the importance assigned to exploratory activity in haptic perception, very few experimental studies have investigated the exact nature of the interaction between stimulus property, exploratory movements and perception. The one question that seems to be prominent is if and how an observer's percept can follow directly and in a quantitative way from properties of movements of the arms or characteristics of skin deformations that result from the hands touching the stimulus. The observation that observers apply a characteristic exploratory procedure that depends on the stimulus property, as illustrated above, may still permit a wide variation in sensory stimulations within this exploratory procedure. A more stringent condition would be that stimulus properties are encoded parametrically in certain properties of arm movements or skin deformation patterns, and that perception follows directly from observing these properties.

Let's assume that, given a certain manner of exploration, some of these properties indeed stand in a fixed relation to the stimulus dimension to be perceived. These properties then form a set of haptic cues. They relate the physics of the environment (the stimulus) to the characteristics of the sensory apparatus. As such, cues represent information about the stimulus in a format that is presumably more directly available to the sensory system. For example, properties of arm movements are resulting forces on limbs and their constituents or kinematic parameters of arm movements; properties of skin deformations are related to stress and strain patterns of the skin. The challenge is to identify and measure these cues for exploratory activity under active-touch conditions. The next step would be to determine which cues correlate best with subjective estimates as obtained in a psychophysical experiment: These would then be the effective cues used by the observer.

Simply outlining this research program is, however, much easier than its practical implementation. Let alone the technical difficulties associated with measuring arm movements and concomitant skin deformations under conditions of free exploration, another problem is that the relation between the stimulus property and the exploratory movements is by no means a one-way interaction. As I have just argued, when exploring an object, the observer is searching for the appropriate sensory stimulations. Movements are the result of a reciprocal interaction between movements initiated by the observer and the sensory stimulations resulting from contact with the stimulus. Of all the movements made, there will presumably be only a subset of movements, unknown to the experimenter, on which the observer eventually focuses attention most to actually perceive the stimulus.

Perceptual biases provide insight into haptic cues

When investigating whether observers' perceptions follow directly from movement characteristics, it is interesting to look for perceptual biases. Whereas threshold measurements mainly tell what information sources are used, biases provide indications on how the perceptual system processes the information. Perceptual biases come in many flavors. In the research described in this thesis, I conducted discrimination experiments or I compared the results from different tasks among each other. In other words, I compared an observer's percept under one experimental condition with an observer's percept under another condition. Perception is biased when the particular stimulus properties in both conditions are the same but the resulting perceptions are different. Throughout this thesis, I tend to distinguish between two types of causes of perceptual biases. The primary focus is on perceptual mechanisms working on a peripheral level. However, this does not mean that I deny the existence of biases that originate at a cognitive level; at places, I touch upon them but my main interest here is in the former category.

A first cause of perceptual biases is related to the transformation of a physical stimulus property to movement profiles or skin deformations. The manner of exploration of a haptic stimulus is typically influenced by the biomechanical constraints of the motor apparatus. For example, if the hand is moved along a certain path in space, the movement is a complex combination of joint rotations. Biomechanical constraints possibly affecting movement include: different moments of inertia for tracing convex and concave pathways because different parts of the arm are involved (cf. Chapter 4); less easily rotating the hand along a concave pathway compared to a convex pathway due to anatomical constraints (cf. Chapter 5); the same path in space involving a different set of joint rotations for the left or right arm (cf. Chapter 2). If these biomechanical constraints then result in systematic differences in movement profiles or skin deformations between two conditions, the particular stimulus property may be perceived differently and thus perceptual biases result. Biases are the consequences of motor errors: Actual exploratory movements do not match movements as intended by the observers, because they are unable to account for the biomechanical constraints of the motor apparatus.

Related to the influences of biomechanical constraints are the constraints set by the response method. In Chapter 3, I investigated haptically straight lines in four different tasks. The focus here is not so much on perception of object properties, but on the spatial relations between objects. The different results for the various tasks must be due to the different exploratory movements as these are forced upon the observer by the specific task. To summarize, biomechanical constraints of the motor apparatus or constraints related to the task may cause perceptual biases. If so, by observing and analyzing arm movements and investigating which systematic differences correlate with perceptual biases, one would get insight into what cues determine perception.

The next step of an observer's percept following from observed characteristics of movements or skin deformations is by no means trivial. These properties are initially coded in activation of receptors in skin, muscles, joints, and tendons. Activations are modulated and processed at various stages of the peripheral and central nervous system. In addition, concomitant activation principally unrelated to the stimulus property under consideration must be discounted. This represents a second source of perceptual biases. If determining properties of arm movements or skin deformation patterns in two conditions are the same but interpreted differently, biases again occur. I am typically referring to this type of perceptual bias when talking about properties being *mis*perceived. For example, in Chapter 6 observers discriminated lengths of linear extents that were either empty or filled with intermediate stimulations. Movement time, which had been shown to be a primary measure of perceived length, was the same for both types of extents, and intermediate stimulations were essentially non-informative about the extent's length, but I still found biases. In Chapter 4, I argue that curvature effects could possibly have been due to speeds along convex surfaces being perceived differently from speeds across concave surfaces, although radii of curvature were the same.

Haptic shape perception: From local to global shape?

A psychophysical investigation of shape perception is not possible without a useful description of shape. The building blocks of a sensible shape theory are contingent on the specificities of the modality. For the visual modality, for example, occluding contours that separate the visual parts of an object from its non-visual parts are important shape features. In case of the haptic modality, the fingers of the hand typically touch and explore small parts of an object successively. Therefore, a good starting point for the haptic case would be to describe shape in terms of features of the local contact surfaces between shape and hands touching the object. From a single contact, actually two contact surfaces arise (Figure 1A). One could investigate either the contact surface on the shape itself or the contact surface on the tip of the finger. By focusing on one of the two, the problem of shape perception is approached from different perspectives.

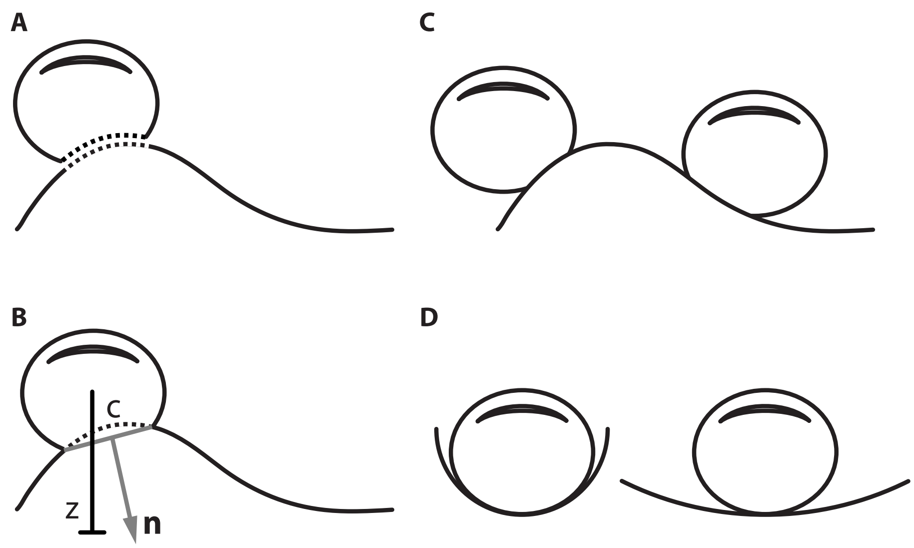

Figure 1.1. (A) The contact surface between object and finger with respect to the object or the finger. (B) Geometric properties of the local contact surface: position (*z*), orientation (**n**), or local curvature (*c*). (C) Shift of contact across the fingertip. (D) The size of the contact surface on the fingertip depends on the shape of the object. Figure B was kindly provided by Maarten Wijntjes.

When investigating the contact surface on the shape itself, the primary focus is only on the geometrical features of the object at the local contact. No reference is yet made to the way in which objects are actually explored. Examples of local shape features include the position of the local contact surface, its average orientation, or the change of orientation over the local contact surface. The last cue is also called local curvature. See Figure 1B for an illustration. More extensive discussions of these cues are given in Chapters 2 and 5.

One could also investigate haptic shape perception by focusing on the contact surfaces on the fingers (Hayward, 2008). With this approach, one explicitly relates local object shape to properties of the contact surface on the scanning finger. Therefore, the relative movement of object and hand has to be taken into account.

The explicit assumption here is that perceiving shape is based on observing cues related to arm movements or skin deformations. Examples include the shift of the contact surface across the fingertip when the finger is tracing the surface of an object or the specific shape and size of the contact area on the fingertip (Figure 1C and 1D, respectively). Chapter 5 will elaborate on this topic.

In focusing on local shape features, one implicitly assumes that integration of these local features at some point forms a necessary step in perceiving precise shape. The suggestion is made that shape perception is a process in which local shape features of adjacent surface patches touched by the fingers are combined to constitute an observer's shape percept. As far as I know, the only direct piece of evidence in favor of this assumption is provided by Pont et al. (1998). They show that the primary geometric cue for dynamic curvature discrimination is the slope difference over the surface. A recent study by Van der Horst and Kappers (in press) more explicitly addressed the role of local contact information in perceiving three-dimensional shape. Observers discriminated circular from elliptic cylinders and square from rectangular cuboids by grasping and scanning the objects. For both objects, kinesthetic cues related to the posture of the hand were available, but for the cylinders there was the additional source of curvature information. Interestingly, the results suggest that performance was primarily determined by this latter source.

In the two chapters of this thesis that deal with haptic shape perception (Chapters 2 and 5), I tend to think about shape perception along the lines just described. However, I do realize that it cannot be the full story. The question is up to what scale this scheme of integration of local shape features is tenable. The advantage of this approach is that it recognizes the importance of local contact information, which is characteristic for the haptic modality. However, by introspection we must also recognize that at some point in the transition from local shape to overall shape, global shape cues take over. When touching a complicated object with both hands, we usually first get an impression of its overall shape, the impression of precise shape coming at a later stage. An example of a global geometric shape cue would be the circumscribed ellipsoid of the particular object, because a common way of touching small objects is to enclose them in one's hands. However, few psychophysical researches have been done on global shape cues for real three-dimensional objects. Studies like the one by Norman et al. (2004) that use natural but complex shapes (e.g., bell peppers) may provide valuable indications for further identification of quantitative shape cues.

Overview

I will now present a brief overview of the investigations presented in this thesis.

Chapter 2 provides an extensive study into bimanual curvature discrimination thresholds and biases. When researching haptic perception, one encounters the problem of a lack of basic quantitative knowledge many times. There is still a strong need for developing a larger body of data on perceptual performance. In this study, I found, among other things, biases indicating overestimations of curvatures presented either on the left or on the right hand. Interestingly, these bimanual discrimination biases were highly idiosyncratic. I argue that biases are possibly caused by slight differences in movements of the left and right arm. Consequently, the idiosyncratic behavior would then be due to natural variations in exploratory movements between observers. Since we find idiosyncratic behavior many times in our haptic experiments with real stimuli, it would be interesting for future research to quantify and explain natural variations in exploratory movements and to understand in what way they affect observers' percepts.

Chapter 3 questions the viability of the concept of haptic space. Here, haptic space is understood to be (1) the set of observers' judgments of spatial relations in physical space, and (2) a set of constraints by which these judgments are internally consistent with each other. I ask observers to construct straight lines in a number of different tasks. I show that the shape of the haptically straight line depends on the task used to produce it. Because of the task-dependence of an elementary notion like a straight line, doubts are cast on the usefulness of the concept of haptic space for providing a general framework: Specification of the response method is apparently needed to define the straight line.

In Chapter 4, I show that when observers trace curved pathways with their index finger and judge distance traversed, their distance estimates depend on the geometry of the paths. In addition, I show that a kinematic mechanism must underlie this interaction: (1) the geometry of the path traced by the finger affects movement speed and consequently movement time, and (2) movement time is taken as a measure of traversed length. Intriguingly, this study provides the mirror version of a study by Pont et al. (1999), which showed that length affects curvature perception. As tracing with the index finger is also a common exploratory procedure for perceiving precise shape, it would be interesting for future research to further investigate the supposed interaction between perceived shape and perceived distance or size.

The study presented in Chapter 5 addresses the question of how kinematic properties of exploratory movements affect perceived shape. It is one of the first studies to do so for free exploration and for real stimuli (as opposed to virtual or rendered stimuli). Here, shape perception is understood in terms of observing certain invariant relations between shape parameters and kinematic properties of the exploratory movements made. I argue that biases in perceived radius are due to systematic differences in exploratory movements for concave and convex strips. As such, this study presents an interesting example of an observer's shape percept directly following properties of the induced arm movements. In the Epilogue to Chapter 5, I discuss in more detail the connections with the findings of the preceding chapter.

Finally, in Chapter 6, I investigated the haptic filled-space illusion for dynamic touch. In the haptic filled-space illusion, observers move their finger across a tangible grating, a punctured edge and so on. Those filled extents are generally judged longer than empty extents, similar to the visual filled-space illusion. I was interested in this illusion given my earlier research on length perception. Since length perception is primarily based on a kinematic mechanism, the question is whether the increased stimulation at the fingertip for the filled extents distorts perception of kinematic variables of speed or time. However, only few studies investigated this illusion. Largely unknown are the parameters affecting the strength of the illusion. In addition, in all of these studies overestimations of filled extents could well be due to so-called end point effects. If that were the case, the illusion would not be a proper spatial illusion. In this study, I controlled for end-point effects and investigated the effect of filler density and movement speed on the strength of the illusion.

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Van der Horst BJ, Kappers AML in press Using curvature information in haptic shape perception of 3D objects. *Experimental Brain Research*

Chapters 2 and 3 have been published:

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Sanders AFJ, Kappers AML (2007) Haptically straight lines. *Perception* **36**:1682-1697

Part of Chapter 4 has been accepted for publication (pending revisions):

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Chapter 5 (excluding the Appendix) and Chapter 6 have been submitted for publication.

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Chapter 2

Bimanual curvature discrimination of hand-sized surfaces placed at different positions

Abstract

This study explores bimanual curvature discrimination of cylindrically curved, hand-sized surfaces. The setup was designed so that the postures of the observers' left and right arms and hands were the same as if the observers were holding a large object in their hands. We measured psychometric curves for observers who used active, dynamic touch; curvatures ranged from 1.18 to 4.05 m^{-1}. Bimanual discrimination thresholds were found to be between 0.26 and 0.38 m^{-1} on average; they were in the same range as unimanual thresholds reported in previous studies. Variation of (1) the horizontal distance between the stimuli or (2) the position of the setup had no effect on thresholds. In addition, we found that a number of observers showed discrimination biases in which they judged two physically different curvatures to be equal. Biases were of the same order of magnitude as the thresholds and could be either positive or negative. These biases can possibly be explained by small differences in left and right arm movements, an explanation that is supported by the position-dependence of biases for individual observers.

Introduction

Haptically acquired information about the shape of an object, the texture of its surface, its temperature, or its weight plays an important role in everyday manipulation of objects. Although we acknowledge that visual feedback most of the time makes a significant contribution, in many everyday actions such feedback is absent. Even with visual feedback present, afferent information can sometimes be essential, as is illustrated, for example, in the case study of a man suffering from a severe peripheral sensory neuropathy (Rothwell et al., 1982). Recently, a growing number of psychophysicists have turned to the topic of haptic perception of object properties. This article focuses on the perception of shape.

One valuable approach to the question of object properties that play a role in haptic shape perception is the use of natural and complex shapes. One could use familiar objects (as did e.g., Klatzky et al., 1985), but perhaps more interesting are (haptically) unfamiliar objects, because of the absence of prior knowledge. What are the physical properties that an observer uses as cues to perceive an object's shape? Although there is a long history of vision research into this question, only few investigators have addressed it in haptics, mostly from the perspective of comparing the two modalities.

Norman et al. (2004) explored visual and haptic shape perception doing experiments in which observers had to compare the shapes of bell peppers. In one of the experiments, a bell pepper shape was manipulated haptically and matched to one of twelve visually presented shapes. Inspection of the objects that had initially confused observers the most suggested that these objects had a similar global shape but differed in what one might call local features, such as "widths of trough gaps […] or large differences in surface curvature" (p. 347). The apparent increase of performance over time would then indicate that these local features gradually came into play and helped in matching the shapes. In any case, these results show that observers were able to mentally represent an object's overall shape, as well as the shape of characteristic substructures. Furthermore, their ability to quickly construct a representation of the overall shape without attending to local features suggests the use of global shape cues. However, the question of the nature of these cues remains.

The stimuli that Lakatos and Marks (1999) used (wooden objects modeling crystal structures) were categorized a priori by the experimenters on global and local geometrical shape features. Observers had to assess both visually and haptically the "overall degree of similarity between the shapes of […] two objects" (p. 897). In one of the experiments, the researchers investigated the effect of exploration time on similarity judgments both for pairs of objects that had similar global shapes but distinctive local features and for objects that differed in global shape and had no local features. Objects from the first set were judged to be more similar over time, whereas similarity judgments remained constant for the second set. Contrary to Norman et al.'s (2004) findings, these results suggest that local shape features are initially weighted more heavily, because they appeared to have obscured the object's global shape.

This contradiction could probably be resolved by considering the characteristic differences between the stimulus sets. However, it should also be noted that the distinction between local and global shape features is somewhat ambiguous. For example, the additional local features in Lakatos and Marks's (1999) research were geometrically defined, but they could also interact perceptually with an object's global shape, as indeed appeared to be the case for some objects.

The most important thing these studies show, however, is that we still have little understanding of the physical object properties that observers may use as cues for perceiving shape in the first place. In both studies, only a sophisticated guess (based on visual inspection of the stimuli by the experimenters, and maybe observers interviews) could be made as to what the determining features were that an observer used to compare the shapes (although admittedly this was not the primary objective of the studies). Garbin and Bernstein (1984), for example, took a more systematic approach to this question. They had participants sort into groups a set of irregular shapes made from clay. The results of a multidimensional scaling analysis were then correlated with the ratings that subjects gave on a number of physical object properties. Psychological measures were taken rather than physical measures, because of the difficulty of precise object quantification for complex forms.

Knowing the cues that play a role is one thing, but determining their psychophysical function is another. If observers indeed compare the curvatures of corresponding parts to match two shapes, what curvature differences are needed to allow the observers to perceive their dissimilarity? How large can the width of a trough gap be before the subject perceives its presence or before it starts changing the perceived overall shape? Because little haptic research has been devoted to psychophysical measurements of three-dimensional object properties (for a review, see Klatzky and Lederman, 2003), we consider these important first steps to be taken when studying haptic shape perception, with the main challenge, of course, of physically quantifying the cues.

In our research, we therefore take a different approach to investigating the perception of object properties: By using stimuli that are mathematically well defined, we try to understand the important physical cues. The strength of our approach lies in our stimuli being real (physical) rather than virtual renderings (see, e.g., Henriques and Soechting, 2003), which allows us to exploit the full range of cutaneous and kinesthetic abilities in a way similar to everyday exploration of natural objects. For reasons that will be explained below, curvature can be considered an important shape parameter. In this research, we mimicked everyday conditions as much as possible by looking at bimanual exploration, setting no restrictions on the way participants explored the stimuli, and designing the setup so that the postures of the observers' arms and hands were the same as if they were holding an object in their hands.

Many objects we encounter in daily life are bounded by smooth surfaces. Even when an object is not completely smooth, its surface usually consists of smooth parts that are interrupted by ridges and edges. For any smooth object, we can approximate the local surface around any of its points in terms of doubly curved surfaces (Koenderink and Van Doorn, 1992). Such a doubly curved surface is fully defined by the 'curvatures' along two orthogonal axes. Since these curvatures are independent of the position of the object in space, they make good candidates for an intrinsic description of shape. We can then characterize the shape of any three-dimensional object in a sufficiently small neighborhood by specifying the two orthogonal curvatures. For that reason, curvature is an important parameter in the study of haptic shape perception.

Consider the example of a sphere (e.g., a basketball). The two orthogonal curvatures of the local surface patch are both equal to the reciprocal of the radius of the sphere. This is intuitively clear, for it is easily understood that the surface of a sphere has a constant curvature that decreases for spheres with larger radii. In this article, we deal with curved surfaces of circular cylinders. Like the sphere, any point on the surface of such a cylinder has the same local curvature, which makes it a suitable object for experiments on curvature perception. The curvature of the surface along the cylinder axis is zero, whereas the curvature in the perpendicular direction is equal to the reciprocal of the radius. In a sense, one might call this a one-dimensional curvature. Note that curvature is not only a measure of shape, but also of size: Most people would say that a wine bottle and a beer bottle have the same shape, although the surface of the beer bottle has greater curvature.

A number of psychophysical studies have investigated unimanual curvature discrimination: Observers had to discriminate between two curved profiles by touching them with one finger or hand successively. Discrimination thresholds were measured for different curvature profiles, stimulus dimensions, and placements on the hand, and for static and dynamic touch. So far, however, little attention has been paid to bimanual curvature discrimination, in which observers discriminate two curvatures by touching them with two hands simultaneously. For example, Kappers and Koenderink (1996) reported that in experiments with cylindrically curved surfaces, performance was better for unimanual than for bimanual discrimination. However, they did not measure psychometric curves and gave no values for discrimination thresholds. In our research, we will focus on bimanual discrimination, but first we will give a short overview of research on unimanual curvature discrimination that is relevant to our study.

One of the first studies to report systematic measurements of unimanual discrimination thresholds was by Gordon and Morison (1982). Observers explored small strips of lengths varying from 2 to 4 cm with the tip of their index finger. For example, the authors found that observers could discriminate between two 2-cm strips with base-to-peak heights of only 0.12 mm and 0.22 mm. More extensive research was done by Pont et al. (1997). They investigated static discrimination of cylindrically curved, 20-cm-long strips placed in nine different positions on the palmar and dorsal sides of the hand. Discrimination thresholds were found to be significantly higher in the dorsal condition. For all positions on the palmar side of the hand, thresholds ranged between 0.22 m^{-1} and 1.48 m^{-1} (reference curvatures were –0.8, 0 and +0.8 m^{-1}). Pont et al. (1997) argued that because cutaneous resolution is much lower on the dorsal side, the result indicated that cutaneous stimulation makes an important contribution to the discrimination of curved strips.

In a follow-up study, Pont et al. (1999) compared static and dynamic touch and determined the effective stimulus for unimanual curvature discrimination. Discrimination was tested under different conditions for cylindrically curved strips of lengths varying from 5 to 20 cm. A surprising result was the quantitative similarity of thresholds for static and dynamic touch, despite the additional component of self-controlled exploratory movements in dynamic discrimination. They also found similar mechanisms underlying both static and dynamic curvature discrimination.

Discrimination performance in the experiments by Pont et al. (1999) could be explained by an attitude comparison model with an incomplete correction for stimulus length. For example, in the case of static touch with one finger, participants discriminated successively presented stimuli by comparing the local slopes. When observers used two or three fingers, attitude differences between the two outermost fingers were compared. Larger attitude differences then corresponded to more curved strips. In the case of dynamic touch, the effective stimuli were again attitude differences. If curvature discrimination were based only on attitude comparison and the distance between the fingers were ignored, observers would underestimate the curvature of shorter strips in comparison with longer strips with the same curvature. Indeed, Pont et al. (1999) found that observers made systematic errors, but these errors were not as large as expected. Apparently, curvature judgments were not completely independent of stimulus length.

Another relevant study concerning unimanual discrimination was conducted by Louw et al. (2000). They determined discrimination thresholds (discrimination from flat) for Gaussian-shaped profiles covering a wide range of spatial scales. The length of the strips varied from 4 to 90 cm. Over the entire range, Louw et al. (2000) found the same dependence of threshold amplitudes on the spatial width of the Gaussian profile; this result is surprising, considering that in this range of scales different cutaneous and kinesthetic mechanisms contribute to haptic perception. Louw et al. (2000) explained that the result is in close agreement with conclusions reached by Pont et al. (1999) concerning the effective stimulus for curvature discrimination.

The present study

The present research focuses on bimanual curvature discrimination. There are a number of questions we hope to answer in this study. First, we want to know how unimanual and bimanual discriminatory sensitivities relate. Threshold measurements enable us to compare these two conditions. One could hypothesize that given the sequential nature of unimanual exploration, bimanual discrimination should outperform the unimanual condition. On the other hand, bimanual discrimination requires the integration of bilateral nerve signals in the central nervous system, which might pose a disadvantage for the bimanual condition.

It is natural for observers to use two hands instead of one when handling objects that are relatively large. In this study, we therefore used hand-sized stimuli. We oriented the stimuli in such a way that the postures of the observers were the same as if they were holding an object in their hands (see Figure 1). They were free to explore the stimuli in whatever way they preferred, and the stimuli were large enough for them to use the entire hand to touch the surfaces. The next question, therefore, concerned the effect of surface area. Depending on the exploration strategy applied, an increase in surface area might give an observer increased skin contact area (thus, additional cutaneous information) and induce additional arm movements up to the shoulder (additional kinesthetic information), relative to the exploration of curved strips. In the case of hand-sized surfaces, thresholds either decrease or remain constant.

Although the shape of an object is independent of its position, it might be more difficult to perceive shape when an object is not right in front of the observer but is held to the left, to the right, or even above the head. It might also be more difficult to perceive the shape of two objects that are farther apart. In this study, we asked ourselves whether the positioning of the stimuli influences discrimination performance. We built a setup with which we could vary the positions of the two stimuli in front of the observer independently on a horizontal line from left to right. We varied the distance between the stimuli, keeping them positioned symmetrically with respect to the observer (Experiment 2), and we varied the position of the stimulus set, placing it to the left, in front, and to the right of the observer (Experiment 3).

The study by Kappers and Koenderink (1996) mentioned above also reported that the placement order of the stimuli had an effect on discrimination in the bimanual condition. For three out of four observers, discrimination performance was much better when the more curved surface was placed on either the left or the right. Which placement order resulted in best performance depended on the observer. A response bias was ruled out as a possible explanation for the observed asymmetry, because both the bimanual and the unimanual conditions would then have been influenced to the same extent, which was not the case. Kappers et al. (1994) reported a similar finding.

The placement order effect as reported in these two studies provides strong evidence that bimanual curvature discrimination may be biased. Suppose, for example, that an observer overestimates the curvature that is presented to the right hand. If in a bimanual discrimination experiment the right stimulus is (physically) more curved, discrimination improves, because the observer perceives an even larger curvature difference between the two stimuli. Correct response scores go up, resulting in a shift of the psychometric curve. In the reverse order, however, discrimination performance is worse. The final motivation for the present study is, therefore, to investigate whether bimanual discrimination can indeed be biased. Does the left or the right hand systematically overestimate curvature? The studies by Kappers and Koenderink (1996) and Kappers et al. (1994) suggest that different observers can have different behavior, since some showed and others did not show a placement order effect. What is more, for different observers in those experiments the effects were in opposite directions. However, it should be noted that these studies were carried out with only a few observers.

General methods

Stimuli

The stimuli in this study had a length and width of 29 cm and a constant peak height of 5 cm. They were made from polyurethane foam filled with an artificial resin. The top surface was cylindrically curved. The reciprocal of the radius of the corresponding cylinder was taken to be the curvature of the stimulus. We produced stimuli with a radius of curvature between 24.7 cm and 84.7 cm; this covered the range of curvatures that was physically possible with these stimulus dimensions. Consecutive stimuli differed by a constant increase in radius of 50 mm. The stimuli had the following curvatures: 1.18; 1.25; 1.34; 1.43; 1.55; 1.68; 1.83; 2.01; 2.24; 2.52; 2.88; 3.37, and 4.05 m^{-1}. Two stimuli were available for each curvature. In Figure 1A, the most and least curved test stimulus are shown as viewed from the side.

Experimental setup

Observers were seated behind a small table and blindfolded to prevent them from seeing the stimuli. We put a heavy metal support with a rail and two metal stimulus holders on top of the table, the center of the rail being about 30 cm from the table's edge. The stimulus holders could be moved independently on the rail in front of the observer from left to right and vice versa. Stimuli were easily fixed onto and removed from the two stimulus holders. The curved side of the left stimulus pointed outwards to the left, the curved side of the right stimulus to the right; the cylinder axis of the stimulus was oriented horizontally. A small hole in the middle of the table enabled the observers to check their positions during the experiments. Figure 1 shows three pictures of the setup, with the stimuli in different positions in each of them. Note that in the experiments curvature differences were much smaller than shown in panel A; we used stimulus pairs like the one shown in panels B and C.

Figure 2.1. An observer trying to feel which of the two surfaces is more curved. Panel A shows the setup in Experiment 1; panels B and C show two examples of setups in Experiments 2 and 3, respectively. Note that in panel A, the most curved and the flattest stimulus are shown, but in the experiments curvature differences were much smaller.

Procedure

The experiments were designed according to the two-alternative forced-choice method. In each trial, two stimuli were presented at the same time: The left stimulus was touched by the left hand, the right one by the right hand. The observers were asked to indicate which stimulus they judged to be more curved and were allowed to answer only "Left" or "Right". A trial started after the experimenter had put both stimuli into position. The participants were free to explore the surfaces in whatever way they preferred, but they were not allowed to touch other sides of the stimuli than the curved surfaces. In effect, all observers used their entire hands, including the palms, to scan the stimuli by moving them systematically up and down over the surfaces. The only exception was observer MH, who used only her fingers to explore the stimuli. No observer used static hand positioning exclusively. Some observers participated in more than one experiment, which they performed in a random order.

Analysis

For each combination of stimuli, we computed (1) the curvature difference, defined as the curvature on the left (which was either the reference or the test stimulus) minus the curvature on the right (either the test or the reference stimulus), and (2) the fraction of all ten repetitions in which the observer judged the left stimulus to be more curved. Cumulative Gaussian distributions were used as psychometric functions and were fitted to the data on a linear scale by means of a maximum-likelihood estimate (see the Appendix). A curve consisted of 14 data points, with each one sampled 10 times.

The fitted psychometric curves are characterized by two parameters, which will be denoted by μ and σ (see Figure 2). Parameter μ (mean) represents the curvature difference at which the observer decided in 50% of all trials that the left stimulus felt more curved. A nonzero mean indicated that the observer showed a bias: Two stimuli that had physically different curvatures gave rise to a response score of 50%. Parameter σ is inversely related to the steepness of the curve. We define the discrimination threshold to be the curvature difference that yielded a rise from 50% to 84% in the response score. In that case, the discrimination threshold is represented by σ. Figure 2 depicts a representative example of psychometric curves obtained in this study. We estimated standard errors in fitted parameters μ and σ using a parametric bootstrap procedure (see the Appendix).

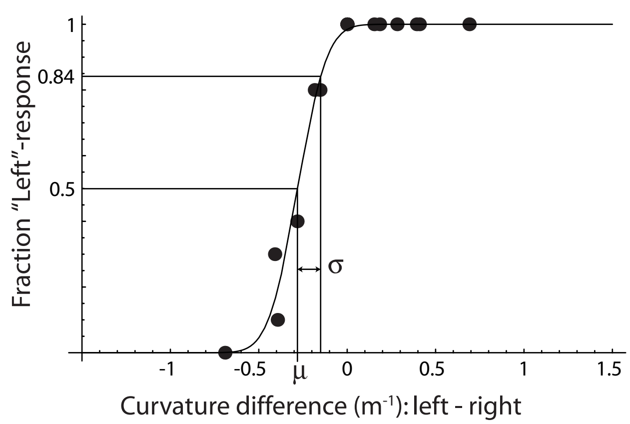

Figure 2.2. The fraction of trials in which the observer responded that the left stimulus was more curved is plotted as a function of the curvature difference between the two stimuli. A cumulative Gaussian is fitted to the data.

Experiment 1

In the first experiment, we measured bimanual discrimination thresholds and biases for three different reference curvatures to find out more about the curvature-dependence of these parameters.

Method

Participants

Ten undergraduate students from the Department of Physics and Astronomy at Utrecht University participated in Experiment 1. According to Coren's (1993) questionnaire, they were all right-handed; three of them were female (CD, DL, and MH). All observers were naive with regards to the aims and design of the experiment and were paid for their efforts. Some observers also participated in Experiment 2 and/or Experiment 3 at an earlier or a later time.

Procedure

We used three reference stimuli with curvatures 1.43, 1.83 and 2.52 m^{-1} and combined each of them with seven test curvatures: Three curvatures were larger than, three smaller than, and one was the same as the standard stimulus. For example, a reference curvature of 1.43 m^{-1} was combined with curvatures 1.18-1.83 m^{-1}. The stimuli were positioned symmetrically with respect to the observer (see Figure 1A); the distance between the two peaks of the stimuli was 30 cm. Reference stimuli were presented to both the left and the right hand; each combination was repeated 10 times. For a single observer this amounted to 420 trials (3 reference curvatures x 7 test curvatures x 2 reference positions x 10 repetitions). Trials were randomized in the following way: All 420 trials were divided into 10 blocks of 42 trials in which each combination was presented once; all blocks were randomized independently. The experiment was divided into sessions of 45 minutes; the entire experiment took about 5-6 h for each observer. Sessions were generally separated by at least one day. The observer received no feedback.

Results

Figure 3 depicts bimanual discrimination thresholds (σ) as a function of reference curvature for all observers in the first experiment. Average discrimination thresholds were 0.26, 0.32 and 0.38 m^{-1} for reference curvatures 1.43, 1.83, and 2.52 m^{-1}, respectively; thresholds ranged between 0.16 and 0.54 m^{-1}. Thresholds and corresponding standard errors correlated strongly: The standard error was 16% of σ, on average. To avoid cluttering the graph, we did not indicate errors in Figure 3.

A one-way ANOVA with a repeated-measures design showed a significant main effect of reference curvature ($F_{2,18} = 15.28$, $p < .001$). To further analyze the effect of reference curvature, we performed paired t tests. Compared to the Bonferroni-corrected significance level (α) of .017, we found that all thresholds differed significantly from each other (0.26–0.32 m^{-1}: $t_9 = 2.95$, $p = .016$; 0.26–0.38 m^{-1}: $t_9 = 4.80$, $p = .001$; 0.32–0.38 m^{-1}: $t_9 = 3.11$, $p = .013$).

The increase of thresholds with reference curvature suggests that curvature discrimination is subject to Weber-like behavior. Therefore, we computed Weber fractions by dividing thresholds by reference curvature to see if they were constant. The average fractions were 18%, 17%, and 15% for reference curvatures 1.43–2.52 m^{-1}. Although a one-way ANOVA with a repeated-measures design did reveal a significant effect of curvature ($F_{2,18} = 5.05$, $p < .025$), paired t tests, with Bonferroni correction lowering the significance level α to .017, indicated that only the fractions 18% and 15% differed significantly (18%-17%: t_9 = -0.55, p > 0.5; 18%-15%: t_9 = -3.00, p = .015; 17%-15%: t_9 = -2.55, p = .03).

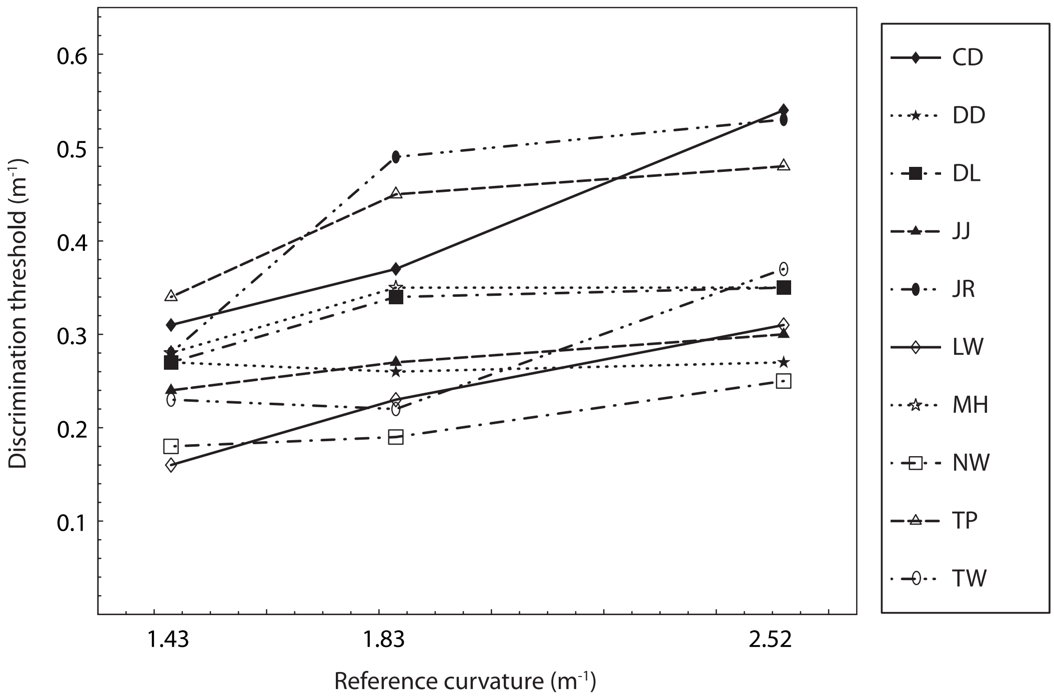

Figure 2.3. Discrimination thresholds against reference curvature for all 10 observers participating in Experiment 1.

Figure 4 gives separate graphs for each observer, representing biases (μ) and corresponding standard errors. Close inspection of Figure 4 reveals that for some observers (e.g., NW) standard errors were small relative to biases: Their biases were significantly nonzero. For other observers, a zero bias fell within one standard error (e.g., DL). Some participants showed positive biases (e.g., CD), and others showed negative biases (e.g., TW). We did not analyze the results in more detail because it was apparent from Figure 4 that biases were strongly participant-dependent. Finally, it should be noted that for a few individual observers, biases seemed to depend on curvature.

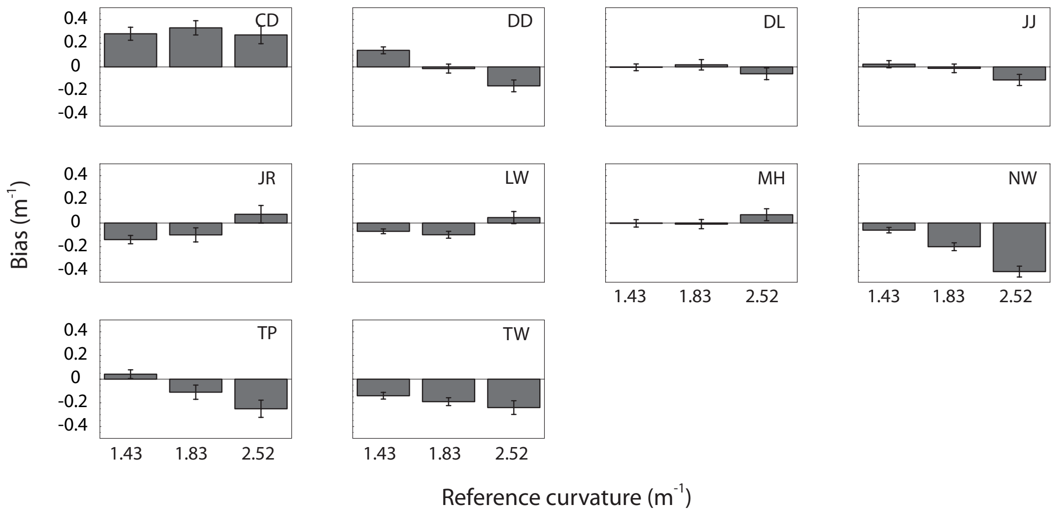

Figure 2.4. Biases against reference curvature (Experiment 1); standard errors are indicated.

Experiment 2

In Experiment 2, we varied the distance between the two stimuli, keeping them positioned symmetrically with respect to the observer. In one trial, postures of left and right hand and arms were the same.

Method

Participants

Ten undergraduate students from the Department of Physics and Astronomy at Utrecht University participated in this second experiment. Four were men (BL, DD, LW, and WG); only CR was left-handed according to the Coren (1993) questionnaire. The observers were all naive and were paid for their efforts. Some observers also participated in Experiment 1 and/or Experiment 3 at an earlier or a later time.

Procedure

The procedure was the same as in the first experiment. This time, however, we used only one reference stimulus, with curvature 1.83 m^{-1}, combined it with seven test stimuli (1.43-2.52 m^{-1}), and varied the peak-to-peak distance. The peak-to-peak distance is defined as the distance between the peaks of the two stimuli. The stimuli were always positioned symmetrically with respect to the observer. We measured psychometric curves for four peak-to-peak distances: 10, 30, 50 and 70 cm (Figure 1B shows the setup for a peak-to-peak distance of 70 cm). This amounted to 560 trials for each observer: 1 reference curvature x 7 test curvatures x 2 reference positions x 4 distances x 10 repetitions. The trials were randomized in the following way: All 10 repetitions were randomized independently and measured successively.

One repetition consisted of four sublists of 14 trials, one for each separation distance. Within each sublist, all 14 trials were randomized; the order in which the four sublists were presented was also randomized. The entire experiment took about 6-7 h for each observer to complete.

Results

The psychometric curves for one particular observer did not fit the data well, so we excluded her results from our analysis.

Figure 5 shows thresholds as a function of the peak-to-peak distance for 9 observers. The overall average discrimination threshold was 0.30 m^{-1}; thresholds ranged from 0.15 m^{-1} to 0.57 m^{-1}. Standard errors were approximately 16% of σ. As expected from Figure 5, a one-way ANOVA with a repeated-measures design revealed no significant main effect of peak-to-peak distance on thresholds ($F_{3,24} = 1.65$, $p > .10$).

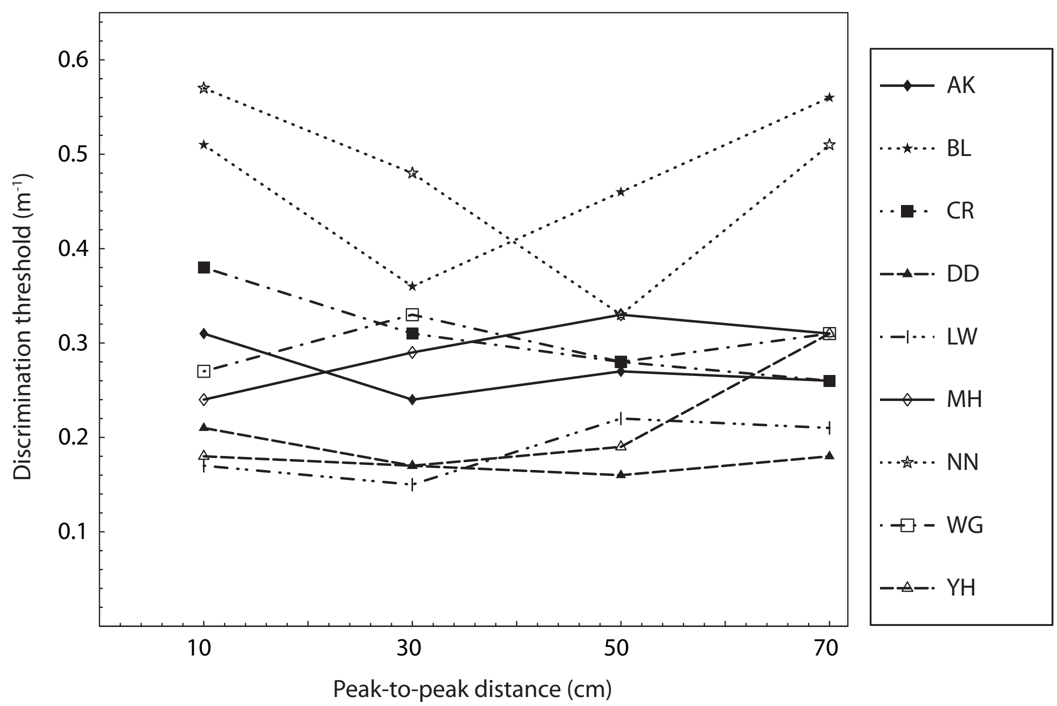

Figure 2.5. Discrimination thresholds against peak-to-peak distance for all nine observers in experiment 2.

In Figure 6, we have plotted discrimination biases against peak-to-peak distance for every observer separately. For most observers, errors were small compared with biases (e.g., DD, YH). Observer BL showed mainly zero biases. Other observers had a nonzero bias for a particular peak-to-peak distance, whereas the bias disappeared for other peak-to-peak distances. It is clear from Figure 6 that there was no general dependence of biases on peak-to-peak distances. Note that for some observers graphs suggest that biases increased or decreased with peak-to-peak distance.

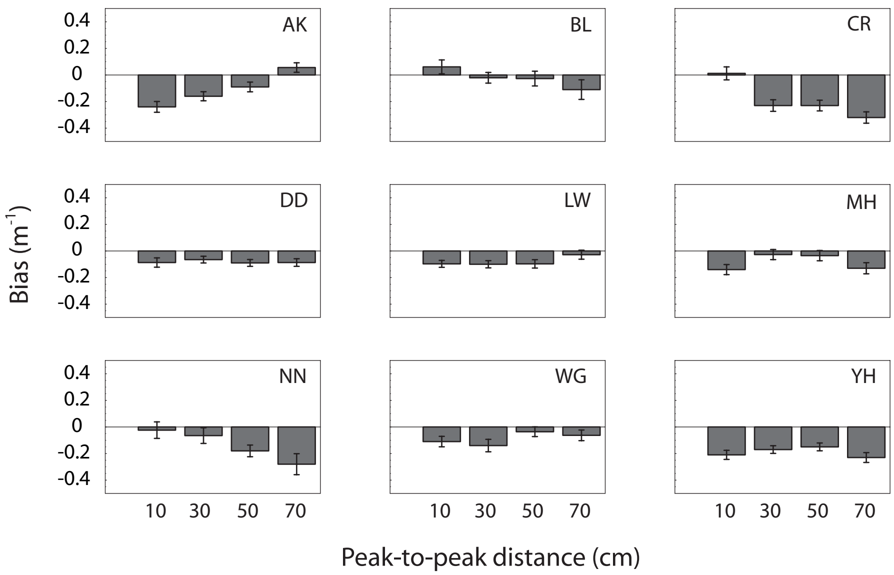

Figure 2.6. Biases against peak-to-peak distance (Experiment 2); standard errors are indicated.

Experiment 3

In Experiment 2, stimuli were always positioned symmetrically with respect to the observer. In the third experiment, we wanted to investigate whether an asymmetrical positioning of the stimuli, causing different postures of left and right arms in one trial, influences performance.

Method

Participants

Ten students from the Department of Physics and Astronomy at Utrecht University participated in this experiment. None of them was left-handed according to the Coren (1993) questionnaire, and four were women (BP, MH, NN, and YH). All participants were naive as to the designs and aim of the experiment. Some observers also participated in Experiment 1 and/or Experiment 2 at an earlier or a later time.

Procedure

The procedure was similar to the procedure in Experiment 2. We combined a reference stimulus with curvature 1.83 m^{-1} with seven test stimuli (curvatures 1.43-2.52 m^{-1}) but kept the peak-to-peak distance between the stimuli at 30 cm. The center point of the stimulus set (the point between the two stimuli) was positioned

20 or 10 cm to the left of the observer, in front of him or her, or 10 or 20 cm to the right of the observer. Positive center positions correspond to a positioning of the stimuli to the right of the observer; negative to the left. Figure 1C shows the setup for a center position of 20 cm. Each observer did 700 trials (1 reference stimulus x 7 tests x 2 hands x 5 center positions x 10 repetitions). Trials were randomized in the following way: All ten repetitions were randomized independently. A repetition consisted of five sublists of 14 trials, one for each center position. Within each sublist, all 14 trials were randomized; the order in which the five sublists were presented was also randomized. Each observer took about 7-9 h to complete the entire experiment.

Analysis

Probably because the psychometric curves for observer NN were very steep, we did not succeed in fitting the curves using a maximum-likelihood procedure. For this observer, we used a least-squares method.

Results

Figure 7 depicts discrimination thresholds as a function of center positions for ten observers. The overall average discrimination threshold was 0.22 m^{-1}; thresholds ranged from 0.02 to 0.40 m^{-1}. We did not indicate standard errors; they were approximately 16%. As expected from Figure 7, a one-way ANOVA with a repeated-measures design showed no significant effect of center position ($F_{2.19,\ 19.73}$ = 1.14, p > .25).

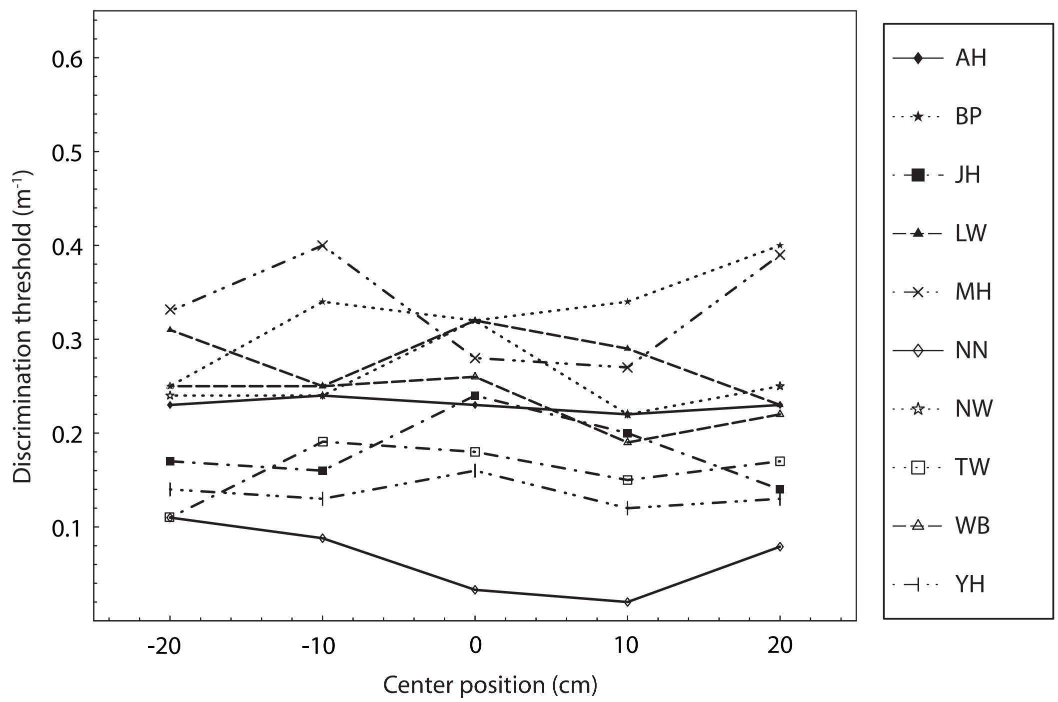

Figure 2.7. Discrimination thresholds against center position for all 10 observers in Experiment 3.

Figure 8 shows biases and standard errors for each observer. The results for most observers are very clear: Some had nonzero biases (e.g., JH, TW), others had zero biases (e.g., AH, NN). Obviously, there was no general dependence of biases on center positions. For several observers, biases seem to have increased or decreased with center position (e.g., NW, WB).

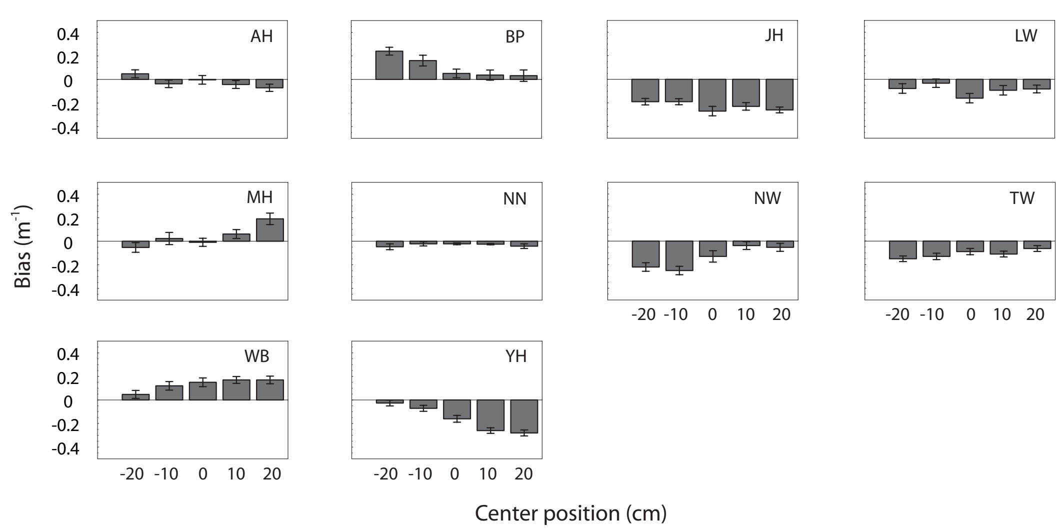

Figure 2.8. Biases against center position (Experiment 3); standard errors are indicated.

Experiment 4

If an observer's response score is 50%, we assume that the observer perceives no curvature difference between the two stimuli. For that reason, the mean of the psychometric curve is sometimes appropriately called the Point of Subjective Equality. However, it is possible for an observer to show a response bias, which means that response is biased towards one of the response alternatives, irrespective of the stimulus intensity (MacMillan and Creelman, 1991). Consequently, the subjectively equal curvature pair then corresponds to a response score that is either above or below 50%, depending on whether the response is biased toward "Left" or "Right", respectively. In Experiment 4, we did matching experiments with five observers who had participated in one or two of the previous experiments. It was the observers' task to match the reference curvature with a test stimulus that they perceived as having the same curvature. This procedure enabled us to measure the subjectively equal stimulus pair directly.

Method

Participants

Five observers participated in this experiment, all of whom had been observers in one or two of the previous three experiments: BL (Experiment 2), JH (Experiment 3), AH (Experiment 3), TW (Experiment 1 and 3), and YH (Experiment 2 and 3).

Stimuli and experimental setup

The stimuli and the setup were the same as in the previous three experiments.

Procedure

All three discrimination experiments had one common condition--that is, a reference curvature of 1.83 m^{-1}, a peak-to-peak distance of 30 cm, and a center position of 0 cm. In Experiment 4, we determined the subjectively equal stimulus pair for the same experimental condition. At the beginning of a new matching series, we informed the observer on which side we had positioned the reference stimulus. A trial started after the experimenter had put both the reference and the test curvature in position. After each trial, the observer asked for a more curved or a less curved test stimulus. This process continued until the observer decided that both the reference and test curvatures felt the same. There were no time constraints. Each of the two reference positions was sampled six times. We picked the first test curvature randomly from the stimuli with curvatures 1.25-3.37 m^{-1}; the order of all 12 trials was also randomized. It took the observer 45 minutes to 1 h to complete the experiment.

Analysis

For every observer, we calculated both the mean and the standard deviation of the curvature differences (left minus right) between the matched stimuli.

Results

For each observer, Figure 9 depicts the bias found in the matching experiment against the bias measured in the discrimination experiment for the same experimental condition. For the two observers who had participated in two discrimination experiments, we computed the weighted average of the two biases. Standard errors are also indicated in Figure 9. The dashed line is the $y = x$ line; points on this line represent observers who had exactly the same bias in both the matching and the discrimination experiment. Observers JH, TW, and YH all had nonzero, negative biases in the discrimination experiments. The first two also showed nonzero biases in the fourth experiment, but for observer YH a zero bias fell just within the standard error. Observers AH and BL showed zero biases in the matching experiment as well as in the discrimination experiment.

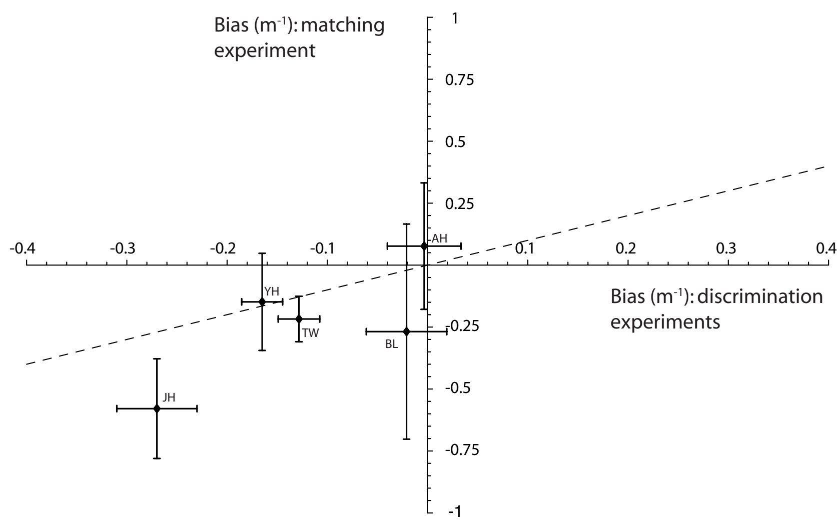

Figure 2.9. Biases measured in the matching experiment (Experiment 4) against biases measured in the discrimination experiments for five observers. Standard errors are indicated.

Discussion and conclusions

We measured curvature discrimination thresholds and biases for hand-sized stimuli and varied curvature and spatial location in four experiments. Observers combined both tactile and kinesthetic sensations from active exploratory movements in a way similar to everyday manipulation of objects. We will now discuss the results in more detail.

Discrimination thresholds

Close inspection of Figures 3, 5, and 7 shows that thresholds were in the same range for all observers. We found a significant increase of thresholds with reference curvature in the curvature range from 1.43 to 2.52 m^{-1}, suggesting that bimanual discrimination is subject to Weber-like behavior. Consequently, we would expect Weber fractions (18%, 17%, and 15% on average) to be constant, but in a statistical analysis we could not decide whether this was indeed the case. Goodwin and Wheat (1992) reported thresholds for much larger curvatures and, consequently, much smaller stimuli. They studied passive discrimination of spherically curved surfaces applied to the finger pad. Their thresholds (at 75% correct) were 18% for a reference curvature of 154 m^{-1}, and 13% for 286 m^{-1}. Although the studies were conducted under different conditions, Goodwin and Wheat's (1992) results together with ours indicate that Weber fractions might indeed be similar across a wide range of curvatures.

Bimanual versus unimanual discrimination

One of the motivations for this study was to compare the results for bimanual discrimination with unimanual curvature discrimination thresholds reported in previous studies. Although limited data on unimanual discrimination are available, we know of two studies for which the experimental conditions resemble those of our experiment. Both studies measured thresholds for hand-sized stimuli that were being touched with the entire hand. Vogels et al. (1996) reported curvature detection thresholds for spherically curved stimuli, and these thresholds averaged 0.23 m^{-1} over three participants. Vogels et al. (1999) used doubly curved surfaces and asked observers to discriminate from flat the curvature along a specified orientation. Discrimination thresholds were between 0.20 and 0.32 m^{-1} on average for seven observers. In these studies, observers used static touch only, whereas in our experiment stimuli were explored dynamically. However, Pont et al. (1999) found that, at least for discrimination of curved strips, static and dynamic discrimination thresholds were similar.

Thus we see that, in comparison with the thresholds obtained in our study (0.26–0.38 m^{-1}), unimanual thresholds were in the same range as thresholds in the bimanual condition. We found no indications that unimanual discrimination performance differs from bimanual discrimination performance. The supposed disadvantage of integrating bilateral signals in bimanual discrimination does not outweigh the drawback of the additional memory component in the unimanual case. Apparently, the ability to discriminate is the same whether two curvatures are touched with two hands simultaneously or with one hand successively.

Contact area

Our observers were free to explore the surfaces in whatever way they preferred. However, all of the observers used dynamic touch, and all except for one (MH) used their entire hands to touch the surfaces. The second question we had was whether an increase in skin contact area has an effect on discrimination performance. The results from Pont et al. (1997) provide the best comparison. The thresholds reported for convex strips (length 20 cm, width 2 cm) that were placed perpendicular to the fingers or in an equivalent fashion on the knuckles or palm were between 0.76 and 1.05 m^{-1} on average, which is almost a factor of three larger than thresholds found in our study. In agreement with Vogels et al. (1999), we conclude that observers combine information from several parts of the hand to compare curvatures.

Discrimination on different positions

The third motivation for this study was to investigate the effect of spatial location on discrimination performance. There was no significant effect on thresholds of peak-to-peak distance or of center position. These striking results strongly indicate that the perception of shape is independent of the position of the object. Indeed, when we handle objects in daily life, its position is hardly ever fixed: We toss the object around in our hands, pick it up from the floor, hand it to someone else, or put it on a shelf. To perform these actions correctly, it is important that we perceive the same shape, regardless of the object's position.

The position independence of thresholds enables us to infer, to some extent, the contributions made by the cutaneous and kinesthetic sense. Peak-to-peak distances covered the observer's entire arm span. The minimum peak-to-peak distance was 10 cm, which corresponded to a configuration in which both stimuli were pushed together. A distance of 70 cm was the maximum distance that allowed all observers to reach both stimuli easily. For different peak-to-peak distances, positions and postures of wrists, forearms, elbows, upper arms, and shoulders varied considerably. For example, angles between hand and forearm for all observers were roughly between 205° and 215° (hands bent outwards) at a peak-to-peak distance of 10 cm and between 165° and 175° at a distance of 70 cm. The same holds for center positions: A shift of the stimulus set of 20 cm was the maximum displacement that every observer could easily cover without having to twist their upper body too much. The postures of left and right arms and shoulders were different for fixed center positions, except for a center position of 0 cm. In the case of center positions +20 or -20 cm, the angles between hand and forearm were roughly between 165° and 175° for one side and between 215° and 225° for the other.

In short, over the range of peak-to-peak distances and center positions considerable variation occurred in the postures of left and right arms and hands, resulting in different kinesthetic stimulation. Arm movements might have also varied with stimulus position, but this is more difficult to assess. In addition, it is reasonable to assume that cutaneous stimulation was roughly the same for different positions: We did not observe the same observer using different parts of the hand to touch stimuli located at different positions. Besides, since observers could easily reach both stimuli in all positions, they were not forced to have completely different skin deformation patterns for stimuli at different positions. Note that we distinguish the cutaneous sense and kinesthesia purely in terms of function (Loomis and Lederman, 1986): Although cutaneous mechanoreceptors in the skin might contribute to kinesthetic awareness (e.g., signaling joint position by detecting skin stretch), the function of the cutaneous sense is to provide awareness of stimulation of the outer surface of the body.

The fact that spatial positioning of the stimuli had no effect on discrimination performance then leads us to two possible conclusions: Either (1) kinesthetic stimulation does not contribute to discrimination of curved surfaces or (2) kinesthetic stimulation does contribute to discrimination, in which case a correction is made for different body postures. The studies by Pont et al. (1997, 1999) and Louw et al. (2000) provide support for the first option. Pont et al. (1999) showed that the additional arm movements made in dynamic touch, as compared to static touch, do not result in lower thresholds. Louw et al. (2000) also measured discrimination thresholds, although they did not vary stimulus position or touch mode. Their thresholds were instead measured over a broad range of stimulus dimensions in which there was considerable variation in primarily kinesthetic stimulation. They found the same dependence of thresholds on the spatial width of the Gaussian profile over the entire range of spatial scales.

Discrimination biases

The final question we had was whether bimanual curvature discrimination biases exist. The mean of the psychometric curve is the curvature difference of the stimulus combination for which the "Left"-response score was 50%. An observer whose psychometric curves had nonzero means was said to show a bias. Close inspection of Figures 4, 6, and 8 indicates that several observers showed nonzero biases, whereas other observers obviously had zero biases. (Note that some observers participated in more than one experiment and therefore appear twice or even three times in these figures.) There were also observers whose biases seemed to depend on curvature or position. In addition, biases could be either positive or negative and had different magnitudes.

If an observer's "Left"-response score was 50%, we assumed that the observer had not perceived any curvature difference between the two stimuli. In that case, a nonzero mean indicates that the observer showed a discrimination bias: Two physically different curvatures were perceived as being the same. As such, discrimination biases indicate that left and right hand transform curvature differently. However, note that a biased observer may be equally sensitive to curvature differences as an unbiased observer in terms of discrimination thresholds. It has been reported several times that biases exist in curvature perception. For example, Davidson (1972) showed that blindfolded participants judged concave edges to be straight. In a similar fashion, observers in Henriques and Soechting's (2003) research showed small concavity biases when discriminating curved trajectories that are traced by moving the hand-held, vertical handle of a manipulandum in the horizontal plane. Vogels et al. (1996) reported that the "phenomenal flatness" of a flat surface was nonzero. In a study by Pont et al. (1998), participants touching a symmetrical, doubly curved surface with one hand judged this surface to be asymmetrical. Vogels et al. (1999) found biases for the discrimination of curvature along a specified orientation of a doubly curved surface. They also found that these biases were influenced by the shape of the surface and that the effect was participant-dependent.

Strictly speaking, observed biases could also be explained by a simple response bias. Irrespective of the stimulus level, the observer's response would then be biased toward one of the response alternatives, and in that case his or her response score for the subjectively equal stimulus pair would be either above or below 50%. This bias would result in a shift of the psychometric curve, just as in the case of a discrimination bias. However, we think it unlikely that biases can be explained by response biases. The studies by Kappers et al. (1994) and Kappers and Koenderink (1996) both give strong indications for discrimination biases. Results of these two studies cannot be explained by response biases. In addition, four out of the five observers who participated in Experiment 4 had similar results in the discrimination experiments. The difference between the discrimination experiments and the matching experiment is that, in the latter, response biases are explicitly ruled out. In conclusion, although we cannot rule out response biases for individual observers, the results show that participant-dependent, bimanual discrimination biases do exist.

Origin of discrimination biases

Now that we know that bimanual discrimination can be biased, the question arises as to the cause of this bias. A study by Kappers et al. (1997) suggests that it probably has its origin at a central level of the nervous system. In their experiments, observers had to match the curvature of a haptically presented surface with a visually presented cross section. The systematic scaling differences between the haptic and visual perception of curvature that observers displayed were similar for both hands. This suggests that if left and right hand touch the same curvature under the same conditions, the signals from the two hands arriving at central levels of the nervous system are probably the same.

However, alternative explanations that focus on the peripheral nervous system are also possible. Remember that observers generally make upward and downward scanning movements. Suppose that the left or the right hand systematically scans the surface over a shorter distance. This difference in scanning length between left and right hand may cause an asymmetrical curvature perception. Pont et al. (1999) and Louw et al. (2000) showed that unimanual discrimination of curved strips is based primarily on the recording of slope differences. In the bimanual case, this would mean that observers record the slope difference over the surface presented to one hand and compare it to the slope difference over the surface presented to the other hand. If a stimulus is scanned over a shorter distance, the observer records a smaller slope difference, resulting in an underestimation of curvature compared to the same stimulus scanned over a larger distance. On the basis of this attitude comparison model, we have computed scanning-length differences between the two hands that would correspond to typical values of biases (see Table 1).

	Reference Curvature (m^{-1})		
Bias (m^{-1})	1.43	1.83	2.52
0.2	3.6	2.9	2.2
0.3	5.1	4.1	3.2

Table 2.1. Typical values of biases and corresponding scanning-length differences between the two hands (in centimeters). Assuming that a given reference curvature was touched over the entire length of its curved surface, we first computed the slope difference over that surface. Given a certain discrimination bias, we could then compute the length of the scanning path for the subjectively equal curvature that resulted in the same slope difference. The scanning-length difference is the difference between the two scanning paths.

Scanning-length differences for biases observed in our experiments are well within the range of the stimulus dimensions. It is not easy to assess whether systematic differences in scanned areas actually existed, because the observers touched the surfaces in various ways. However, the seeming dependence of biases on spatial position in the case of individual observers may provide some evidence. If for different spatial positions observers make slightly different arm movements, the scanning length can depend on spatial position. More generally speaking, if scanning-path lengths indeed play an important role in curvature perception, there would be an effect on everyday exploration of objects, when observers seem to make more random movements over the surfaces. However, we must bear in mind that not only attitude differences but also curvature plays an important role in curvature discrimination and that of course curvature is the same for different scanning lengths.

As we made clear earlier, it is desirable that more psychophysical research into the haptic perception of real-object properties be done. This paper adds to the growing body of quantitative knowledge of haptic shape perception.

Appendix

We fitted the psychometric curves to the data by means of a maximum-likelihood fitting procedure. The likelihood is the probability that a set of observations will occur, given a particular set of parameters. By maximizing the likelihood, we find the best-fit parameters. Contrary to the least-squares method, this procedure takes into account the binomial distribution of the data. The likelihood is the product of the chances of each individual data point.

In the case of a two-alternative forced-choice experiment, the chance P is binomially distributed with the psychometric function ϕ giving for each stimulus pair x_i the probability of a correct response: $p_i = \phi(x_i|\mu,\sigma)$. If n_i is the number of times the ith point on the psychometric curve has been sampled and y_i is the number of "Left"-responses for curvature difference x_i, the likelihood then becomes

$$L(\mu,\sigma) = \prod_{i=1}^{m} \binom{n_i}{y_i} p_i^{y_i} (1 - p_i)^{n_i - y_i}$$

Maximization of this function yields best-fit estimates of parameters μ and σ. Finally, we estimated standard errors using a parametric bootstrap procedure (Wichmann and Hill, 2001). We simulated 100 random data sets using the original best-fit parameters μ and σ, fitted psychometric curves again, and calculated the standard deviations of the collections of μs and σs. These standard deviations were taken as the standard errors for both parameters.

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Chapter 3

Haptically straight lines

Abstract

In this research, we set out to investigate haptically perceived space. Large deviations with respect to physical space have already been shown to exist. Here, research on haptic space is continued by investigating straight lines constructed by touch. In four experiments, subjects were asked to produce straight lines between two reference markers that were in the horizontal plane at a fixed distance from each other. Each experiment corresponded to a different task: two different interpolation tasks, an intersection task and a pointing task. Straight lines had an orientation that was approximately frontoparallel. Observers used both hands; manipulation was unrestricted. Although we found considerable differences between observers, the overall pattern of results showed that haptically straight lines were generally curved away from the observer. However, in one of the interpolation tasks they corresponded to physically straight lines. In addition, the pointing task generally produced larger deviations than the other three tasks. Taken together, the results show that there is no unique definition of the straight line, a conclusion that questions the viability of the concept of haptic space.

Introduction

In marked contrast to the large number of studies dealing with visually perceived space, very little attention has so far been given to haptic perception of spatial relationships. Blumenfeld (1937) was probably the first to embark on this project. One of the things he investigated was the shape of apparently parallel lines, which he called "alley curves". He had blindfolded subjects straighten out small threads attached to needles at different positions in the horizontal plane so as to make them parallel to each other and the midsagittal plane. Diverging and converging patterns of the apparent parallels were observed, depending on their distances from the midsagittal plane.

Since then, a number of researchers have been dealing with the haptic modality and have shown that, when other geometrical properties are perceived, distortions with respect to the physical reality occur too. For example, when the distance between two points was estimated by tracing a tangible straight-line path (a raised-line drawing, for example), a line segment having a radial orientation relative to the body was found to be judged longer than the same line segment having a tangential orientation (e.g., Armstrong and Marks, 1999; Cheng, 1968; Day and Wong, 1971; Heller et al., 1997). When, instead of tracing a straight-line path, subjects take a detour in going from starting point to endpoint, they make large errors in estimating the distance between the two points; these errors are not constant but vary with the length of the detour taken as well as with the distance between the endpoints itself (Brambring, 1976; Faineteau et al., 2003; Lederman et al., 1985). Kappers and Koenderink (1999) studied collinearity in the horizontal plane by asking subjects either to rotate a small bar to point at a target or rotate two bars to lie along the same line, and they studied parallelity by having observers rotate the two bars to be perceived as parallel. They found significant deviations from physical collinearity and parallelity, and they argued that the results were consistent with each other, suggesting a common spatial representation underlying all three tasks. Furthermore, Lakatos and Marks (1998) showed that haptically explored angles consisting either of wooden blocks or raised lines tend to be overestimated. Fasse et al. (2000) suggest that distortions in angle perception and length perception are geometrically inconsistent. Participants moved the handle of a robot arm inside a virtual rectangle of varying dimensions in the horizontal plane. In agreement with the radial-tangential effect mentioned above, a square container was perceived to be an elongated rectangle. However, when asked to judge the relative magnitudes of the angles between the diagonal and the two sides, their estimates again indicated perceptual distortions but in an opposite direction. Finally, there is a large body of literature on haptic illusions (e.g., Gentaz and Hatwell, 2004; Heller et al., 2002; Millar and Al-Attar, 2000, 2002; Suzuki and Arashida, 1992), as well as on haptic bisection tasks, in which subjects generally err to the left of the veridical center of a horizontal line (for a review, see Jewell and McCourt, 2000).

Here, we will focus on what is haptically perceived as a straight line in a number of different tasks. A few studies suggest that systematic deviations are to be expected in the perception of straightness. Blumenfeld (1937) did an experiment in which observers had to put "the middle of three rods into the phenomenally pectoral parallel plane determined by the two outer ones". He found that observers systematically positioned the middle rod too far away, producing a 'phenomenal plane' that was concavely shaped with respect to the body. However, it is unclear how the observer interpreted this task: It is contradictory to describe a plane as being parallel to the chest, as the observer might not think of his chest as being flat, especially not with the dimensions of Blumenfeld's (1937) setup (the distance between the two outer rods was 42 cm). The question is whether we would still find these deviations if the observer was simply asked to put the middle rod into the plane (or onto the straight line) determined by the other two rods.

The results obtained by Kappers and Koenderink (1999) provide clearer evidence that, at least in their experimental paradigm (pointing and collinearity task), apparently straight lines are curved. Participants performed the experiments unimanually and the deviations strongly correlated with horizontal distance, but were almost zero when the bars and targets were on a vertical line (a line that was parallel to the midsagittal plane). Averaged over three subjects, they found a horizontal gradient of 8° m^{-1}.

Finally, there are a few studies in which the subjective straightness of curved edges was investigated when subjects moved their fingers along them (Blumenfeld, 1937; Crewdson and Zangwill, 1940; Davidson, 1972; Hunter, 1954; Rubin, 1936; for a review, see Appelle, 1991). Edges were flexible rulers that were between 10 and 30 cm long and generally had a frontoparallel orientation. Again, subjectively straight edges were concave towards the observer, their midpoints being displaced between 0.2 and 5.3 mm. However, the manner of exploration was severely restricted in all of these studies. Participants used only one hand and except in the study by Blumenfeld (1937), who is not clear about this point, movements were restricted to rotation either of the forearm about the elbow or of the outstretched arm about the shoulder.

Little research has been done on modelling haptically perceived space, by which we mean that perceptions of different geometrical properties are described by and related to each other within a single model (intrinsic geometry of perceptual space; Todd et al., 2001). We already mentioned the study by Fasse et al. (2000) in this respect. Cuijpers et al. (2003) derived a metric model, which they based on a haptic parallelity task (Kappers and Koenderink, 1999). The model predicts that 'haptic frontoparallel lines' (lines having the same orientation as in our experiments) are always concave towards the observer. However, not only is it important for a model that claims to describe haptically perceived space to incorporate different geometrical aspects; it is also essential that the perception of a single geometrical property does not depend on the task that is used to reveal the subject's percept.

In visual research, however, there are clear indications that task-dependences can occur. For example, Koenderink et al. (2000, 2002) let observers construct visually straight lines ('frontoparallels'). Two experiments were performed on a large lawn in broad daylight and observers were allowed binocular vision as well as eye and head movements. Participants either had to rotate a distant pointer using a remote control to aim it at a target (pointing), or they had to direct a radio-controlled vehicle carrying a stake to the intersection point of two line segments, each indicated by a pair of stakes (intersection). In both tasks, apparently frontoparallel lines up to about 1 m from the observer were concave. However, frontoparallel lines further away (up to 15 m) became convex in the pointing task. What is interesting is that the opposite curvatures found in the two tasks cannot be accounted for by a single non-Euclidean structure of visual space.

This raises the question whether different tasks in our haptic experiments will result in similar curvatures. If so, a common spatial representation might be underlying all tasks, which could then tentatively be called haptic space. The next step would be to determine its intrinsic geometrical structure: To investigate to what extent the distortions reported in haptic perception of other spatial properties can be described by the same model. In this respect, it makes sense to investigate the shape of haptically straight lines since in axiomatic geometry a (straight) line is one of the undefined notions that one starts with when developing a geometrical system (e.g., Blumenthal, 1961). In a similar vein to Koenderink et al. (2000, 2002), we set out to investigate the effect of task on haptic perception of straightness. In each of four experiments, corresponding to two different interpolation tasks, an intersection task and a pointing task, subjects had to produce straight lines. If the distortion of haptic perception of physical space depends on the way physical space is being explored, one might well expect the tasks to have different outcomes, although each task shows what the observer perceives as straight. To mimic natural exploration, we gave subjects as much freedom of exploration as possible, given the constraints of the tasks.

Methods

Observers

Ten students at Utrecht University participated in each of the four experiments. All but AB were right-handed according to Coren's (1993) questionnaire; four of them were female (BH, MB, NN, and YH). For all subjects, we measured shoulder width, shoulder height (when seated), upper arm length, forearm length, and hand length: Average values were 42.2 (3.4), 63.9 (3.1), 32.9 (2.1), 28.7 (2.8), and 18.9 (1.2) cm, respectively (standard deviations are in brackets). All observers were naive concerning the aim and designs of the experiments. They were paid for their efforts. One observer was unable to perform the intersection experiment and we replaced her with another observer, using his data instead.

Stimuli and apparatus

In the interpolation and intersection experiments, the setup consisted of a magnetic whiteboard, measuring 120 cm by 90 cm, that was laid on a table. Figure 1 shows a schematic drawing of the setup; the height of the setup was 76 cm. Large, circular magnets (diameter 2.4 cm) served as reference markers indicating the straight line; smaller circular magnets (diameter 1.8 cm) were the test magnets that the observer had to position. These test magnets could be easily moved across the whiteboard without much effort. A large plastic sheet with a coordinate grid printed on it covered the whiteboard to allow for easy recording of the positions of the magnets. A round stool without back or arms was placed at the origin (seat height 50 cm). A short metal bar attached to the front of the table prevented the observers from sliding forward on their chair and touching the table edge with their bodies.

The setup we used for the pointing experiment was the same as that used in Kappers and Koenderink (1999) and essentially the same as the one just described. Instead of a whiteboard, we used a large iron plate with small holes in it. An aluminium bar (length 20 cm, diameter 1.1 cm) had a small pin attached to the middle that could be inserted into one of these holes so that the bar could rotate without being displaced. Subjects had to point the bar, which had an arrow-shaped ending on one side, at a marker, which consisted of a small circular magnet with a diameter of 2.4 cm and a small pin attached underneath to fit into one of the holes. Bar and marker could be placed in exactly the same positions relative to the observer as the reference magnets in the other three experiments. Protractors printed on a plastic sheet covering the iron plate enabled the experimenter to easily register the bar's orientations.

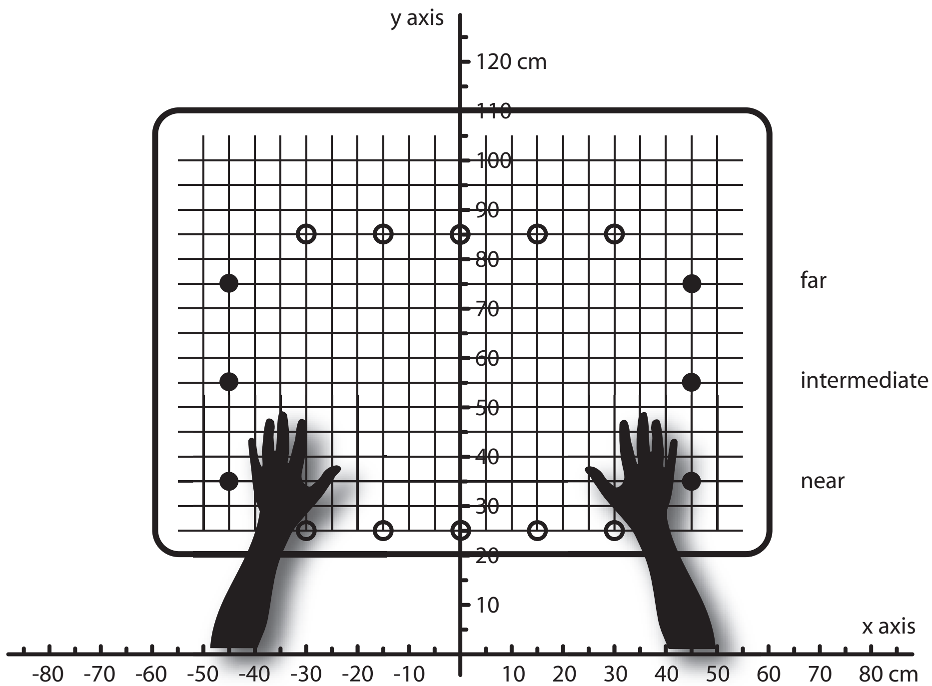

Figure 3.1. Schematic drawing of the experimental setup. The observer is seated at the origin; the *x* and *y* directions are indicated. Closed circles represent positions of the reference magnets or pointer positions. Straight lines were approximately parallel to the front part of the body at either near, intermediate or far distance. The open circles indicate the positions of the reference magnets for the vertically oriented dummy lines (Experiment 3). In the pointing experiment (Experiment 4), a pointer was placed in the same positions as the reference magnets.

In each of the four experiments, observers constructed straight lines between two reference points. These reference points were either magnets (interpolation, intersection) or a rotatable bar and a marker (pointing). The observer was seated at the origin. Coordinates of the two reference points were (±45, 35), (±45, 55), or (±45, 75), so the horizontal distance was 90 cm in all instances and the perpendicular distance between observer and straight line could be 35, 55 or 75 cm. In the remainder of this paper, these distances will be called near, intermediate, and far, respectively. The six reference points covered a fair amount of the workspace in the horizontal plane in front of the observer. We decided to investigate only a single distance between the reference points, because the primary aim of this paper was to compare deviations from physically straight across different tasks. One should therefore be cautious in generalizing the results to smaller and larger extents and the exact dependence of deviations on distance between the reference points remains still to be investigated. However, one might expect that, as the distance decreases, the magnitude of the deviation decreases too, since in the limit case the two points coincide. Kappers and Koenderink (1999) showed that deviations increased with horizontal distance in their pointing and collinearity experiments. In addition, pilot experiments suggested that in case of the other experimental paradigms in this study, deviations increase with horizontal distance too. Therefore, we focused on lines having a frontoparallel orientation and we chose to use the maximum horizontal distance that every observer could easily reach to have deviations as large as possible.

Procedure

To prevent the participant from getting a visual impression of the spatial layout of reference magnets and pointer, we covered the setup with a piece of cloth that was not removed until the subject was seated and properly blindfolded. Furthermore, we instructed observers not to touch the edge of the whiteboard to ensure that they would not use the edge as a cue for physical straightness. Otherwise, manipulation was unrestricted: Observers could move their upper body and use both hands in whatever way they preferred.

We imposed time constraints in all experiments to reduce experimenting time (see below). We had observed in pilot experiments that during a trial, all observers quickly arrived at their approximate final settings, after which they made only small adjustments. The time constraints might therefore have increased only the variability. The experimenter confirmed with the observers that they had enough time to perform the tasks satisfactorily.

The experiments were divided into sessions of 45 to 60 min; on average, it took approximately 3 h for one observer to complete all four of them. The order of experiments was randomized across observers. Every experiment started with a few practice trials. The subjects were never given any feedback on their performance to prevent them from developing any cognitive strategies during the experiments to compensate for deviations. Thus, learning effects did not occur. Some experiments were completed within one session, but others were split into separate sessions.

Experiment 1. Interpolation with five test magnets (IP5)

Subjects were asked to arrange five small test magnets in a straight line between two reference magnets. They were told that they did not need to make the magnets equidistant, but magnets were not allowed to touch each other in the final setting. All observers decided to distribute the five test magnets approximately evenly. We chose to use five test magnets because the average distance between the magnets, if they were put in a physically straight line, would then be 15 cm, which is less than the span of the hand. Thus, when going back and forth between the test magnets, subjects could touch two neighboring magnets with one hand at the same time. In principle, they could use a previously set test magnet as an additional reference and adjust the position of the next test magnet by orienting the hand.

The experimenter first put the observer's left hand on the left reference magnet and the right hand on the right magnet for a few seconds, after which he put the right hand on the test magnets, which were always grouped at (0, 25). A trial started as soon as the observer moved one of the five test magnets. Each of the three distances were measured three times, all in random order. The time per trial was restricted to 1 min.

Subjects used a mixture of different hand and arm movements to perform this task. It is therefore difficult to know from pure observation if there was any particular movement that they relied upon more than others. Generally, observers applied a three-stage procedure: first, they touched the reference magnets for a few seconds; then, they put the test magnets in their approximate positions; finally, they checked and adjusted the positions of the test magnets by repeatedly making two kinds of arm movements. They put their hands on the reference magnets again and either moved both of them towards each other or moved only one of them towards the other hand, adjusting the test magnets on the way.

Experiment 2. Interpolation with one test magnet (IP1)

The second experiment was similar to the IP5 experiment, but instead of five magnets, observers put only one magnet on the straight line determined by the two reference magnets. They were told to put the test magnet approximately halfway between the reference magnets, but not necessarily on the segment's midpoint. This was to ensure that the distance between the magnets was much larger than in the first experiment. In this case, when moving from one magnet to another, the observer's hand had to let go of the first before reaching the second. One might therefore expect larger deviations in this experiment than in the first.

The procedure was the same as in the IP5 experiment. Each distance was measured 10 times in random order; time per trial was restricted to 10 s. Observers applied the same pattern of hand and arm movements as in the first experiment.

Experiment 3. Intersection (IS)

In the third experiment, subjects had to find the point of intersection of two straight lines, each indicated by a pair of reference magnets. Since they were unable to touch all four magnets at the same time, participants had to remember the location of the first straight line when placing the test magnet on the second one. Although they could make repeated adjustments to the test magnet's position, this task might be more demanding than the previous two tasks because of the extra memory component. Indeed, although one observer was able to finish the interpolation tasks, she was unable to perform this experiment, because she appeared to be unable to recollect the positions of the first pair of reference magnets after having felt a second pair, even after a considerable number of practice trials.

Each of the three horizontal lines had to be intersected by five vertical dummy lines. These dummy lines were indicated by two reference magnets positioned at coordinates (-30, 25/85), (-15, 25/85), (0, 25/85), (15, 25/85) or (30, 25/85). Thus, at each of the three distances - near, intermediate and far - we obtained five intersection points. The experimenter first put the observer's hands on the reference magnets of the horizontal line for a few seconds, after which he put them on the reference magnets for the vertical dummy line, the right hand on the nearest magnet. Finally, he put the right hand on the test magnet near (0, 25). A trial began when the observer had started moving the test magnet. Each intersection point was measured three times, amounting to a total of 45 trials for a single observer. The order of trials was randomized; time per trial was restricted to 30 s. Only four reference magnets and one test magnet were placed on the whiteboard during each trial.

Experiment 4. Pointing (PT)

Finally, in the fourth experiment subjects were instructed to rotate a pointer towards a reference marker, subject to the restriction that they did not explore the space between pointer and reference marker. Pointer and target were placed at the same positions relative to the observer as the reference magnets in the first three experiments. This experiment differed from the first three in that subjects did not move their hands between the two reference points.

A trial started as soon as the experimenter had put the observer's hands on pointer and target. Observers were instructed to remove their hands only after they had made their final setting. Each distance was measured 10 times: 5 times with the pointer on the left and 5 times on the right. The order of trials was randomized; time per trial was restricted to 10 s.

Analysis

To be able to compare deviations from physically straight across experiments we needed to devise a single error measure that could reasonably be used to quantify deviations in all experiments. Deviations from physically straight were analyzed in terms of a single curvature parameter k according to the formula:

$$y(x) = -\tfrac{1}{2}k(x - \tfrac{1}{2}h)(x + \tfrac{1}{2}h) + v \qquad (3.1),$$

with h the horizontal distance between the reference points (90 cm) and v the vertical distance between observer and straight line (35, 55, or 75 cm). This formula represents a class of parabolic arcs that connect the reference points and are symmetric around the y axis (see Figure 2). If the curvature parameter k was positive, then the haptically straight line was concave towards the observer (pointing away). If it was zero, the haptically straight line was also physically straight. A negative k indicated that the haptically straight line was convex, that is, pointed towards the observer. The larger the absolute value of k, the larger is the magnitude of the deviation. The dimension of the curvature parameter k is inverse distance. One could roughly interpret k as the inverse of the radius of a circle fitted through the reference points and the corresponding data point.

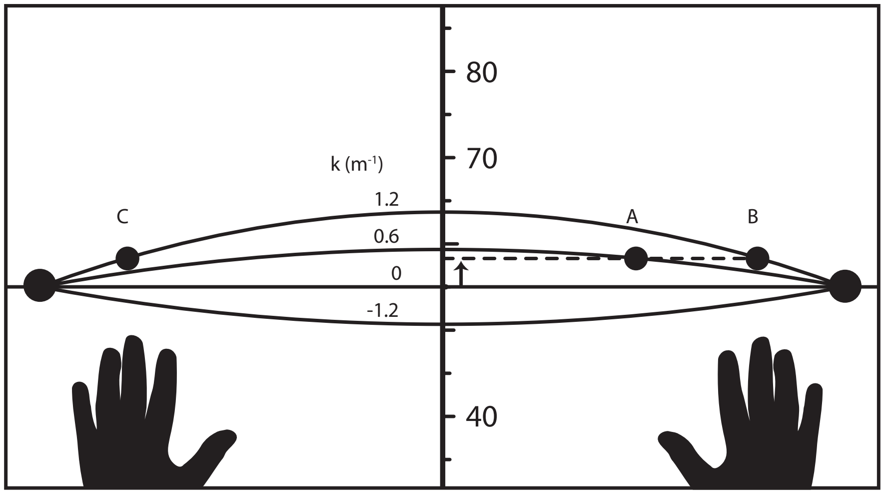

Figure 3.2. Illustration of the curvature parameter *k* that was used to analyze deviations from physically straight. Four parabolic arcs are drawn corresponding to the values of *k* as indicated in the figure. A zero curvature indicates the physically straight line between the two reference magnets. Magnets A and B have the same perpendicular distance (arrow) to the physically straight line but represent different *k*s. Positions of test magnets B and C are symmetric in the *y* axis: these magnets are therefore assigned the same *k*.

For each individual data point (test magnet or pointer orientation), we computed *k* according to this formula. In the first three experiments, (x, y) represents the position of a test magnet and in the pointing experiment, the pointer is taken to denote the tangent to a parabolic arc, thereby uniquely defining the arc's curvature. For each distance, the sample of 15 data points (IP5 and IS) or 10 data points (IP1 and PT) produces an estimate and standard error for *k*.

We consider *k* to be an appropriate measure of the deviation from physically straight. Suppose that an observer positions two test magnets A and B (Figure 2) at the same perpendicular distance from the veridical straight line (arrow), but test magnet B is set closer to the reference magnet on the right. Then it is reasonable to assign a larger deviation to magnet B, as the hand moving from the reference marker to the test magnet produces the same perpendicular distance but travels over a shorter path. In addition, since subjects used both hands, it is also reasonable to assign the same deviation to test magnets B and C that have the same positions relative to the right and left reference magnets, respectively (Figure 2). In other words, the deviation parameter should be symmetric in the *y* axis. Parameter *k* satisfies both properties.

Results

General

Figure 3 depicts representative examples of raw data obtained in this study for each of the four experiments. It can be seen that almost all data points indicate that the apparently straight line pointed away from the observer. However, for observer MB haptically straight lines were sometimes quite close to physically straight, especially when they were at far distance. Figure 4 shows average curvatures per experiment for each observer separately.

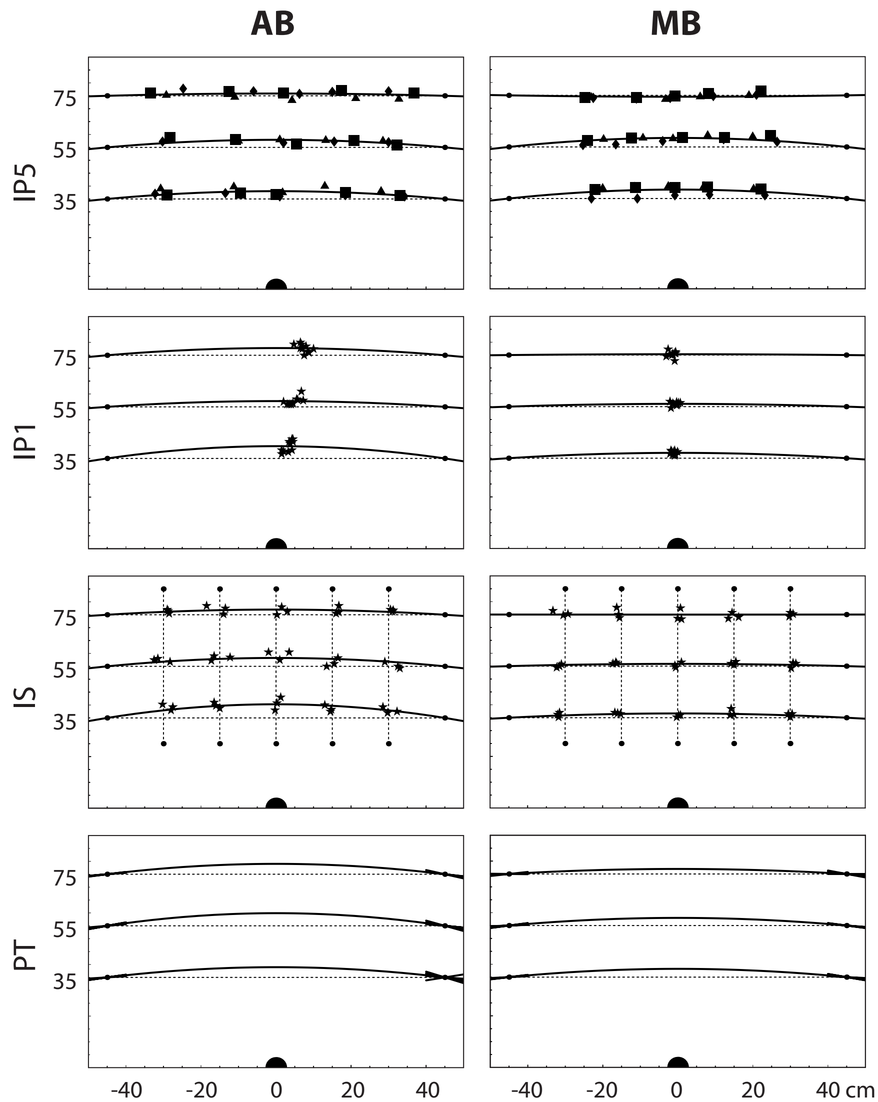

Figure 3.3. Raw data for two representative observers AB and MB. Dashed lines indicate physically straight lines. Distances along the x and y axes are given in centimeters. Interpolation with five test magnets (IP5): different symbols represent different trials.

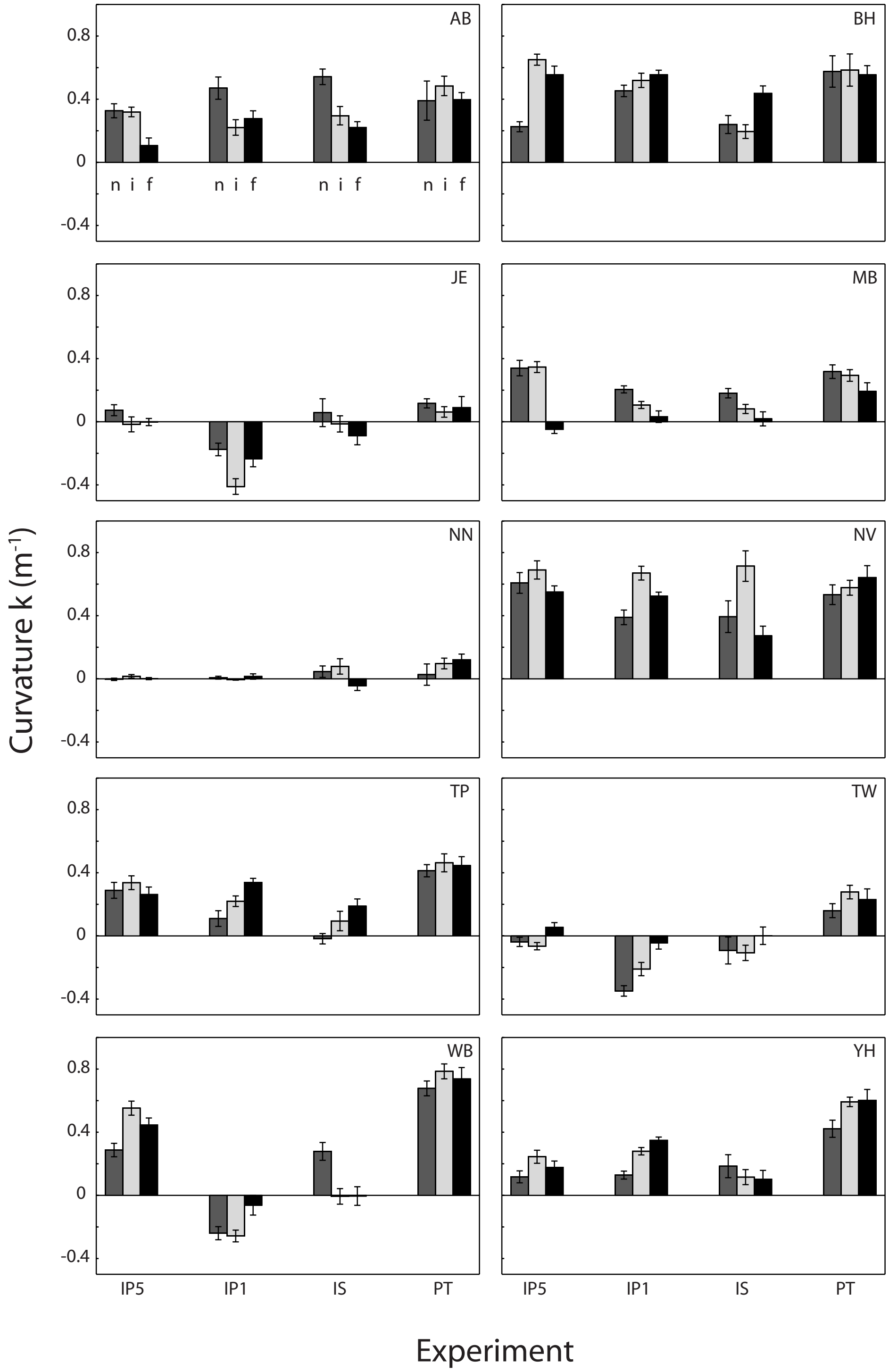

Figure 3.4. Deviations specified for each observer separately. Note the subject-dependence. The three bars for each experiment correspond to near, intermediate, and far distance, respectively.

For every experiment, the results averaged across subjects are shown in Figure 5. Grand means were 0.25, 0.13, 0.15 and 0.40 m^{-1} for IP5, IP1, IS, and PT, respectively. These curvatures correspond to circles with radii between 2.5 and

7.7 m. Simple t tests showed that average curvatures in all experiments except for IP1 were significantly different from zero (IP1: $t_9 = 1.4$, $p > .15$; but IP5: $t_9 = 3.5$, $p = .005$; IS: $t_9 = 2.7$, $p = .024$; and PT: $t_9 = 5.6$, $p < .001$). Thus, on average, subjects' settings deviated from the physically straight line in IP5, IS, and PT.

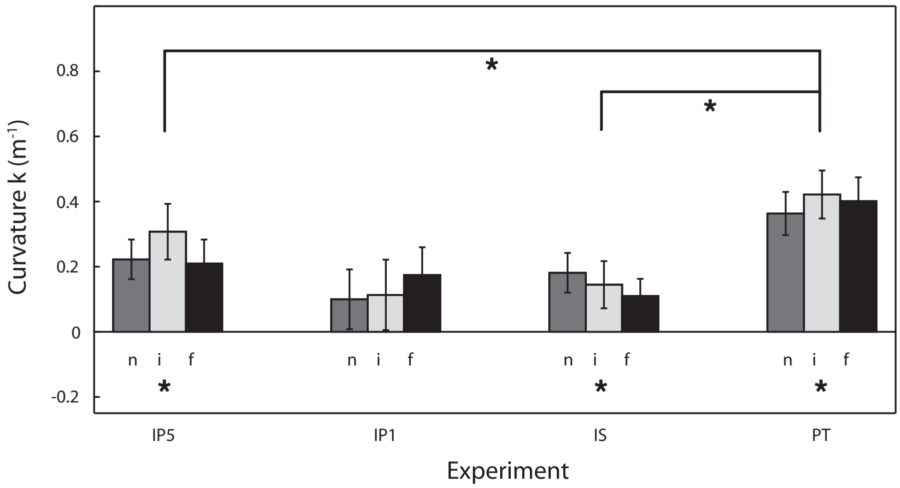

Figure 3.5. Curvature parameter k averaged across observers: If k was nonzero, then the haptically straight line deviated from physically straight. Bars denote curvatures per experiment and per distance (near, intermediate, far); standard errors are indicated. Grand means were 0.25, 0.13, 0.15, and 0.40 m^{-1} for IP5, IP1, IS, and PT, respectively. Only for IP1 was the grand mean not significantly different from zero. Haptically straight lines were significantly more curved in the pointing (PT) experiment than in intersection (IS) and interpolation with five test magnets (IP5).

Next, we tested for an effect of task and of distance on the curvature of the apparently straight line. A two-way ANOVA with a repeated-measures design revealed no significant main effect of distance, but it did show a significant main effect of experiment ($F_{2,18} = 0.4$, $p > .5$; and $F_{3,27} = 8.8$, $p < .001$, respectively).

To further analyze the main effect of experiment, we performed paired t tests with Bonferroni correction lowering significance level α to 0.008. These indicated that PT produced significantly larger deviations than IP5 and IS ($t_9 = 3.9$, $p = .004$; and $t_9 = 4.3$, $p = .002$), but they just failed to show a significant difference between PT and IP1 ($t_9 = 3.2$, $p = .011$), although the difference between the grand means was the greatest for these two experiments. As expected from Figure 5, pairwise differences between IP5, IP1, and IS were non-significant (IP5 and IP1: $t_9 = 1.8$, $p > .10$; IP5 and IS: $t_9 = 2.5$, $p = .033$; IP1 and IS: $t_9 = 0.3$, $p > .5$).

In addition, we computed Pearson's correlation coefficient ($N = 10$) for each pair of tasks. Pairwise correlations for IP5, IP1, and IS were all significant (IP5 and IP1: $r = .69$; IP5 and IS: $r = .80$; IP1 and IS: $r = .87$; all ps $< .03$). The correlation coefficients between PT and IS, and between PT and IP1 were non-significant ($r = .59$, $p = .076$; and $r = .49$, $p = .15$, respectively). This latter result indicates a high level of variability between the curvatures in these two tasks (see Figure 4), explaining why we did not find a significant curvature difference between PT and

IP1. Surprisingly, the correlation coefficient between PT and IP5 was highly significant ($r = .85$, $p = .002$).

Finally, the ANOVA revealed that the interaction between task and distance was significant ($F_{6,54} = 2.5$, $p = .031$). We further analyzed this interaction by conducting one-way ANOVAs for each experiment separately. The effect of distance was non-significant in IP5, IP1, and IS ($F_{2,18} = 2.1$, $F_{2,18} = 1.3$, and $F_{2,18} = 0.8$, respectively; all ps > .15), but it just reached significance in the case of PT ($F_{2,18} = 3.6$, $p = .049$). However, none of the subsequent pairwise comparisons were significant, when compared against a Bonferroni-corrected significance level of 0.017 (near-intermediate: $t_9 = 2.7$, $p = .024$; near-far: $t_9 = 1.4$ and intermediate-far: $t_9 = 1.2$, both ps > .1).

Details

Close inspection of the raw data for IP5 suggested that for most observers the haptically straight line was somewhat flattened in the middle, that is to say the middle magnet was systematically positioned below the parabolic arc. For some observers, it was positioned even closer to the physically straight line than its two neighboring magnets, like for example AB in Figure 3. We investigated whether in this experiment the curvature of the middle magnet was indeed systematically smaller than that of the other magnets. We did this by splitting the curvatures into two groups, one group containing the curvatures of the middle magnet and another containing the curvatures of the remaining four magnets, and testing for a significant difference. Average curvatures were 0.18 m^{-1} for the middle magnet and 0.26 m^{-1} for the remaining four test magnets. A paired t test showed that the difference was indeed significant ($t_9 = 3.9$, $p = .004$).

The test magnet in IP1 and the middle test magnet in IP5 were both positioned halfway between the two reference points. In analyzing the main effect of experiment we found no significant difference between IP5 and IP1. However, it might now be interesting to compare the curvatures of the test magnet in IP1 with those of the middle magnet in IP5, since in this latter experiment the deviations of the middle magnet turned out to be smaller than those of the remaining four test magnets. We hypothesized that if observers used the same kind of arm movements to perform both IP5 and IP1 experiments, the curvatures in the IP1 experiment would be comparable to those of the middle magnet. A paired t test showed that the difference between the curvatures in IP1 and the middle magnet in IP5 was indeed non-significant ($t_9 = 0.9$, $p = .4$).

Furthermore, we tested whether any of the body measures taken correlated with curvature. We computed correlation coefficients between the curvatures per distance and per experiment and each of six body measures: shoulder width, shoulder height, upper arm length, forearm length, hand length and arm length (arm length was derived by adding lengths of upper arm, forearm and hand). All 72 r^2 values were smaller than 0.26 and none of these was significant at the 5% level (all two-tailed p values well above .10).

Finally, we checked whether the results in the PT experiment were symmetric with respect to the pointer positions. We performed paired-samples t tests on the average curvatures per subject for left and right pointer positions, as well as on the standard deviations. Neither the mean nor the standard deviation showed a significant effect of pointer position (t_9 = 0.5, p > .5; and t_9 = 2.0, p > .08, respectively).

Discussion

In four experiments, subjects were asked to construct straight lines at three different distances. Each experiment corresponded to one of the four tasks: interpolation with five test magnets (IP5), interpolation with one test magnet (IP1), intersection (IS), and pointing (PT). Straight lines were in the horizontal plane at waist height and had an orientation that was approximately frontoparallel. The results were analyzed in terms of a curvature parameter k, which indicated the amount of deviation from veridical. Overall, we found that the shape of the haptically straight line deviated from what would be physically straight; in particular, that haptically straight lines were generally curved away from the observer. Only in one of the interpolation experiments, IP1, did we find an overall curvature that was non-significant.

All four experiments presented in this paper concern active haptic perception as Klatzky and Lederman (2003) define it: the interplay of sensory systems (cutaneous and kinesthetic) on the one hand and self-initiated and self-controlled movements of the body parts that elicit the sensations on the other hand. In other words, motor actions are necessary for obtaining sensory information and sensory information in turn is used to guide the motor actions. Active touch is touch as one encounters it in an ecologically valid context. Both components -motor actions as well as sensory feedback- were needed for completing each of the four tasks. Observers initially positioned magnets or rotated a bar. Subsequently, they alternately checked their settings, and adjusted them if needed by making arm movements so that they would only consider the task finished when their settings corresponded to what they felt to be a straight line.

One reviewer wondered whether the deviations we found could really be termed perceptual distortions or whether they actually are motor errors resulting from the movements needed for producing the settings. In this respect, it is important to keep in mind that sensory feedback was available at all times while participants were performing the task. During the final stage of each task the participants checked whether they perceived the arrangements of magnets or bars to correspond to a straight line. As such, the experimental paradigms presented in this paper are true perceptual tasks. As we explained in the Methods, the time constraints that we imposed should not have markedly changed observers' performance. Time limits were chosen such that the observer could still perform the task satisfactorily. Thus, all four tasks show what the observer haptically *perceived* as being straight.

The results from the pointing (PT) experiment replicated those from a previous study on this task by Kappers and Koenderink (1999). If we take the horizontal gradient for each of the three participants in their study to compute curvature scores for a horizontal distance of 90 cm (our setup), the average deviation would be 0.27 ± 0.12 m^{-1}, which is somewhat smaller than the average curvature for the pointing task reported in this study but still well within the range. Participants performed the experiment bimanually in our research and unimanually in the study by Kappers and Koenderink (1999). However, only three subjects participated in the latter study, and therefore we have to be cautious about speculating on an effect of unimanual versus bimanual exploration in the pointing task. Kappers (1999) showed that the unimanual condition caused slightly smaller deviations in the parallelity task. It is suggested that deviations in both tasks are caused by the same underlying mechanism (Kappers and Koenderink, 1999).

Of all the studies on subjective straightness of curved edges that we mentioned in the Introduction, the one by Blumenfeld (1937) qualifies best for comparison with our results. He used the longest edge (30 cm) and it does not seem that he posed any restriction on arm movement, although the edge was scanned only with one finger. The concave bend that was perceived as straight was displaced at the centre by 2.9 mm, corresponding to a curvature of 0.25 m^{-1}. However, scanning of curved edges does not correspond to any of the experiments performed in this research.

The experiment by Blumenfeld (1937) in which participants had to arrange three rods was similar to the IP1 experiment except that the distance between the two reference markers was about half the distance in our setup. Surprisingly, he found a mean displacement of the middle rod of 11 mm (Blumenfeld, 1937: Table 3), corresponding to a curvature of 0.50 m^{-1}, which is about a factor of four larger than the value in our experiment. One would expect curvatures to become smaller when the distance between the reference markers decreases. However, there were substantial differences in experimental procedure (such as single movement direction, use of only right index finger, moving between rods through midair) that preclude us from judging a genuine effect of distance between the reference markers.

The focus of this paper was not to analyze in detail how different constraints on manipulation in a particular experimental paradigm affect the magnitude of the deviations. The main aim was to investigate whether curvatures were dependent on the task when manipulation was restricted as least as possible. In a sense, deviations from physically straight were not task-dependent, as average curvatures had the same sign in all experiments. However, a statistical analysis did reveal overall curvature differences between the four tasks. The general picture emerging was as follows. The four experiments can be split into two groups: the two interpolation tasks (IP5 and IP1) and intersection (IS) on the one hand, and pointing (PT) on the other hand. Average curvatures for the tasks in the first group were the same, while all pairwise correlations were significant. Thus, we must conclude that individual participants produced the same curvatures, which suggests that every participant had a common spatial representation to solve each of the three tasks. Interestingly enough, given the idiosyncratic curvatures as apparent in Figure 4, these individual spatial representations were not identical but were related across observers by a scaling factor. All observers become the same, when this scaling factor is removed.

The relationship between the first three experiments and pointing was somewhat ambiguous. The correlation coefficients between PT and IP1, and between PT and IS were non-significant. Thus, no argument can be given to support a common spatial representation underlying pointing and these two tasks. To our surprise, however, the correlation between PT and IP5 was highly significant, as well as the difference between the mean curvatures. Thus, assuming that participants used different representations to perform both tasks, the two representations were related within observers. Given the similarity of the first three tasks, the finding that pointing was related with only one of them is puzzling. We will discuss this finding in more detail below. At this point, we conclude that we have shown that when observers are asked to haptically construct a straight line, the outcome can depend on the specific task at hand.

To understand the cause of this task-dependence, one would have to analyze in detail the set of manipulations that comprise each task. Since we found that pointing was the odd one out, the manipulations needed to complete the two interpolation tasks (IP5 and IP1) and the intersection task apparently were similar but differed from the operations needed to perform the pointing experiment. Indeed, although we initially designed each of the four tasks to be distinct, a common manipulation for subjects in these three tasks was to move their hands between the two reference markers, whereas in the pointing experiment they were not allowed to do this. Instead, an important factor in the pointing task is the orientation of the hand in space when placed at different positions (Kappers, 2005). The biomechanical constraints of those arm movements must therefore have been a determining factor in the concave shape of the straight line in these tasks: Movement of the hand along a straight path in external space entails a complex combination of joint rotations, the primitives for any arm movement. However, in our experiments subjects used both hands and it remains unclear in what way the two arms interacted. A detailed analysis of the IP5 experiment showed that the curvatures of the middle magnet were significantly smaller than those of the outer four magnets, suggesting that the haptically straight line in this experiment consisted of two arcs, one for the left part of the line segment and one for the right part, which meet somewhere near the body midline. Then, this left arc might correspond to movement of the left arm and the right arc to movement of the right arm. Finally, note that our focus on biomechanical constraints here does not imply that we assume deviations to originate from motor failures (see above).

Interestingly enough, we found that deviations in interpolation with five test magnets (IP5) and pointing were related: A participant producing for example large deviations in the pointing experiment had large deviations in the IP5 experiment as well, although significantly smaller compared to the former. The average distance between the magnets in the IP5 experiment (15 cm) was smaller than the span of the hand and of the same order of magnitude as the length of the pointer used in the pointing experiment (20 cm). We observed that to position a particular test magnet in the IP5 experiment, subjects often applied a strategy of touching this magnet and a previously set neighboring magnet with one hand and then rotating the hand around this last magnet, while holding the other hand on a reference magnet. This procedure closely resembled that of setting the pointer in the pointing experiment, which might explain the strong correlation between the two experiments. Since participants could not use this exploratory procedure in the intersection and interpolation with one test magnet (IP1) experiments, the correlation between pointing and these two experiments should be non-significant, as was indeed the case. Of course, subjects made various other types of arm movements in IP5 as well, such as sweeping one of the hands between the two reference magnets, a procedure they also applied in IP1 and IS. Thus, within participants deviations in the IP5 experiment were smaller than in the pointing experiment, that is to say they moved into the direction of the deviations found in the IP1 and intersection experiments.

We found no general effect of distance between the observer and the straight line. However, the interaction effect suggested an influence of distance in the pointing experiment, although it should have been small, since none of the pairwise comparisons reached significance. This result is in agreement with Kappers and Koenderink (1999) who showed that deviations in the pointing and collinearity task in general correlated only weakly with vertical distance, although for an occasional observer significant slopes were found. The absence of an effect in the first three experiments might seem surprising at first sight: If there is a biasing effect of joint rotations on the shape of the straight line, then one might expect an effect of distance, since at near and far distance participants moved their hands with elbow bent or stretched, respectively. The biasing influence of joint rotations in these experiments, however, was small. Curvatures we found corresponded to circles with radii between 2.5 and 7.7 m, which is much larger that the radius of the outstretched arm. Thus, possible effects of distance would be hard to detect. The absence of any significant correlations between curvatures and body measures such as shoulder width and arm length supports this argument. Finally, we briefly mention that we investigated only a single horizontal distance and it is an empirical question as to how curvatures generalize to smaller or larger extents.

In all four experiments, we investigated what the observers perceived as a straight line. However, the different tasks did not necessarily yield the same curvatures. In addition, the model of Cuijpers et al. (2003) that was based on the results of a parallel-setting task predicted that lines having the same orientation as in our experiments would have a curvature of 0.52 m^{-1}, which is much larger than the curvatures we found. Note, however, that of all four experiments the average curvature in the pointing experiment is closest to the value in Cuijpers et al.'s (2003) model; this is as it should be, since the deviations in the pointing and the parallel-setting task are claimed to stem from the same source (Kappers and Koenderink, 1999). If haptic perception of space depends on the way space is being explored, this has important implications for the notion of haptic space. In particular, we found that subjects have different operational definitions of the straight line, depending on the task at hand. In other words, when speaking of the shape of the haptically straight line, one should specify the set of manipulations or exploratory procedures by which participants arrive at their percept.

It is, of course, reasonable to expect that we will be able to explain the curvature differences that we found by the nature of the different manipulations required for each task. For example, the orientation of the bar with respect to the hand or the relative positions and orientations of the hands with respect to each other may have been misperceived. However, this approach misses the point regarding haptic space, or perceptual space for that matter, that we made in our introduction. We assume that haptic space has an *intrinsic* geometrical structure (Todd et al., 2001): The observer's (mis)perceptions and the relationships between these can be described - to a certain degree - by a formal system (Blumenthal, 1961) without any reference to the physical environment. However, we found that the observer's judgment of straightness cannot be decoupled from the physical context. We do not know yet how to integrate the results of these different tasks into a single geometrical framework. Much in line then with the reasoning of Koenderink et al., (2002), we question the viability of the concept of haptic space.

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Chapter 4

Curvature affects haptic length perception

Abstract

One possible way of haptically perceiving length is to trace a path with one's index finger and estimate the distance traversed. Here we present an experiment in which observers judge the lengths of paths across cylindrically curved surfaces. We found that convex and concave surfaces had qualitatively different effects: Convex lengths were overestimated, whereas concave lengths were underestimated. In addition, we observed that the index finger moved more slowly across the convex surface than across the concave one. As a result, movement times for convex lengths were longer. The considerable correlation between movement times and length estimates suggests that observers take the duration of movement as their primary measure of perceived length, but disregard movement speeds. We further investigated whether these speeds differ because gravity modulates movement speeds across the different surface types, and we considered several mechanisms that could underlie observers' failure to account for speed differences.

Introduction

When estimating distance or size by touch, observers apply a range of different exploration strategies. For example, one could estimate the size of an object by picking it up between thumb and index finger or between both hands and use kinesthetic information about the postures of fingers and arms. Alternatively, small objects can be impressed on the skin of the fingertip to extract size information from cutaneous stimulation. On a different scale, proprioceptive information resulting from the act of walking can be used to infer the distance traveled. In this paper, we will focus on a different, very common strategy for haptic length perception: observers moving their finger or hand along an edge or across a surface to estimate the distance traversed.

Until now, researchers have only investigated linear pathways or curved pathways that were entirely contained in the horizontal plane (see below). No research has so far addressed the general case of length perception on arbitrary surfaces in three dimensions. This is an interesting research topic from the perspective of haptic shape perception, since the shape of a three-dimensional object is essentially defined by a surface bounding a certain portion of space, and since movement of the index finger or hand is a stereotyped pattern of action associated with assessment of that object's shape (Klatzky and Lederman, 2003).

Many geometrical descriptors of shape exist, curvature being one of the more advantageous ones from a psychophysical point of view, because it is independent of the position and orientation of the object. In mathematical terms, any smooth surface can be locally approximated by doubly curved surfaces, being either egg-like or saddle-like (e.g., Hilbert and Cohn-Vossen, 1932). Thus, any shape can be construed as a collection of such elementary surface patches. In this research, we investigated length perception on cylindrically curved surfaces. Participants moved their index finger across such a surface and estimated the distance traversed.

When using limb movement to perceive length, observers' length estimates are closely tied to the kinematic characteristics of the tracing movements. For example, it has been shown for both active and passive movements of the hand that observers judge linear extents traced at higher speeds to be shorter than the same extents traced at lower speeds (Hollins and Goble, 1988; Lederman et al., 1987; Von Skramlik, 1933; Wapner et al., 1967). Movement speeds could differ by as much as a factor of one hundred (Hollins and Goble, 1988). However, it is not simply duration of movement that determines perceived length. For a given duration, considerably longer pathways were still judged longer than the shorter ones. Similarly, when observers actively explore a curved pathway (a raised-line drawing for example) and estimate the straight-line distance between starting point and end point, they systematically overestimate it. These overestimations vary with the length of the detour taken as well as with the straight-line distance between the end points (Faineteau et al., 2003; Lederman et al., 1985).

A related illusion that is important to our research is the so-called radial-tangential effect (RT effect) for whole-arm movements in the horizontal plane: Linear extents oriented radially from the trunk are overestimated relative to tangentially oriented extents (for an overview of literature on the RT effect, see McFarland and Soechting, 2007). In addition to this, researchers have observed that radial movements are executed slightly more slowly than tangential movements and thus take more time (Armstrong and Marks, 1999; Wong, 1977).

According to one hypothesis that has been put forward to account for speed differences, these stem from the inertial anisotropy of arm movements (Wong, 1977). Moments of inertia are usually greater for radial arm movements, in which case they require more energy than movements in the tangential direction. Perhaps observers do not account for differences in moments of inertia when executing arm movements (cf. Gordon et al., 1994), causing them to move faster in the tangential direction. The basis for the RT effect would then be as follows (Marchetti and Lederman, 1983): To use movement time as an estimate of distance traversed, observers try to move at constant speeds, but they do not succeed; the small speed differences that arise are "undetected" (p. 46) by the observers, which causes overestimations of radial extents.

Marchetti and Lederman (1983) tested the inertial anisotropy hypothesis by attaching small weights to the moving hand, thus altering the moments of inertia, and investigating how this affects length estimates. They predicted that the effect of the weights would be that observers move their hands more slowly and that the corresponding increase in movement time would then cause observers to overestimate the length of the movement with weights compared to the length of the movement without weights. It is, of course, debatable whether one could prove that observers do not compensate for inertial anisotropies of arm movements by showing that they do not compensate for differences in external loads. Unfortunately, the results of the experiment by Marchetti and Lederman (1983) were inconclusive, but more importantly, the actual effect on movement speeds was not measured. Therefore, even clear results would have been hard to interpret concerning the origin of the illusion.

If undetected speed differences are the basis for the RT effect, then one might expect the illusion to disappear when speed profiles are made equal along both orientations. McFarland and Soechting (2007) tested this in a length discrimination experiment by having observers hold the handle of a robot arm and manipulating their arm movements. In the active conditions, force fields were generated such that the observer could actively explore a virtual contour. In the passive conditions, the observer's hand was guided along the contour by the robot arm, following a sinusoidal speed profile. Surprisingly, when the observer's hand was being moved with equal maximum speeds in the radial and tangential directions such that movement times were proportional to distances in the same way for both orientations, the RT effect persisted and replicated the illusion in the active control condition. Likewise, when movement times were made constant for all lengths so that the observer would have to rely entirely on speed and acceleration cues for discriminating lengths, the RT effect did not differ from the control condition. In a third passive condition, maximum speeds for the tangential orientation were 35% lower than for the radial orientation, leading to a corresponding increase in movement times for the former direction. Although this was not reported, it seems reasonable to assume that observers noticed the substantial speed difference, but again the RT effect persisted. Thus, radial lengths were overestimated, whereas movement durations were longer for the tangential direction. Finally, McFarland and

Soechting (2007) tested an additional active condition in which an extra resistive force opposing tangential movements was produced. The added resistance made movement times longer in the tangential direction than in the radial orientation, whereas the opposite was the case in the control condition. Once more, however, the RT effect was present in this condition and did not differ from the control condition.

In particular the results for the passive condition that tested equal speed profiles argue against an explanation for the RT effect in terms of undetected speed differences, because in that case one would expect the RT effect to disappear. In our opinion, the experiments conducted by McFarland and Soechting (2007) suggest that an alternative and perhaps more likely explanation for the RT illusion is in terms of misperceived speeds or, possibly, misperceived durations. In other words, there is a systematic bias in perceiving speeds or durations between the radial and tangential orientation, although the cause of this systematic misperception still remains unclear. This point will be further elaborated on in the General Discussion.

In this report, we present a new experiment in which participants make arm movements to estimate distance traversed. Instead of being asked to judge linear extents they are now asked to judge lengths of paths across cylindrically curved surfaces. Because observers use the same exploration strategy in our experiment as in the RT effect, it is reasonable to assume that the same mechanisms for length estimation underlie both experimental tasks. Thus, possible systematic misperceptions of curved lengths are expected to have causes similar to the causes of the misperceptions of linear extents in the RT effect. Questions concerning the basis of possible curvature effects therefore have theoretical implications that generalize to the RT effect, and vice versa. Consequently, the experiments reported in this paper are also of importance for unraveling the RT effect, since the basis for this effect is still poorly understood.

In Experiment 1, we set out to investigate whether observers make systematic errors in perceiving the length of cylindrically curved pathways. Because previous research had shown that length estimates are closely linked to the kinematic characteristics of arm movements, we decided to record movement profiles of the tracing index finger. One of the findings of Experiment 1 is that movement speeds differ for movements along convex and concave pathways. In Experiment 2, we tested whether these speed differences are due to gravity effects. Finally, in the General Discussion we return to the question of the mechanisms that underlie misperceptions of traced lengths.

Experiment 1

In Experiment 1, we investigated whether there are any curvature biases in haptic length perception. We tested for effects of both the magnitude and the sign of curvature, using circular pathways that had a frontoparallel orientation. Thus, we investigated perception of both convex and concave lengths at different radii. In six discrimination experiments, observers compared convex with flat lengths, flat with concave lengths, and convex with concave lengths at two radii. We had no a priori expectations about the existence of curvature biases. However, if curvature biases do indeed exist, we expect their magnitude to increase with curvature: the larger the curvature of the cylindrical surface, the larger the possible curvature biases. Besides measuring observers' length judgments, we also recorded movement profiles of the index finger while it traced the pathways.

Method

Participants

Six right-handed participants in their 20s took part in this experiment (five males, one female). They were naive as to the aims and designs of the experiment. Participants were paid for their efforts.

Apparatus

The setup consisted essentially of (1) a set of steel plates that were either flat or had been milled into cylindrically curved surfaces of fixed radius, (2) a framework in which the plates needed in a particular condition could be easily mounted, and (3) a set of flexible magnetic strips of varying lengths. Figure 1 shows the setup in its upright orientation, as used in Experiment 1, together with its relevant dimensions. All three possible combinations of the convex and the flat plates, the concave and the flat plates and the convex and the concave plates (Figures 1A, 1B, and 1C, respectively) were available at radii of 10 and 20 cm. Note that throughout this paper the terms convex and concave are always defined with respect to the finger touching the surface: In Figure 1, all surfaces are being touched from above, the palm of the hand facing downwards. A simple rail system by which the plates could easily slide over each other allowed for efficient and rapid presentation of the strips. The framework was fixed onto a wooden board (not shown). The participant assumed a fixed position relative to the setup: The setup's midline (bold dashed line) lay in the sagittal plane passing through the right shoulder, as well as in the horizontal plane about 10 cm above the participant's navel. The spatial arrangement of the plates (Figure 1D) was such that the midpoints of all strips were at approximately the same location in space. The purpose of this particular arrangement was to make the posture of the participant's arm as similar as possible across the different surfaces. Magnetic strips had a thickness of 0.6 mm and were cut from a roll of commercially available magnetic foil. The strips lay entirely in the participant's frontoparallel plane. Their midpoints were either on the setup's midline or positioned 1.5 cm to the left or to the right of it, as indicated by numbers 1, 2, and 3 in Figure 1D. The reason for this will be given below. The blindfolded participants were seated comfortably behind the setup. They moved their finger across the strip's edge that was furthest away. The distance between this edge and the frontal edge of the steel plate was 17 cm, which is roughly the distance between the tip of the outstretched finger and the wrist. The magnetic strips' corners were readily felt by the fingertip to indicate the edge's end points. Otherwise participants could freely move their arm and, more particularly, they did not rest their elbow on any support while moving the index finger along the strip. Thus, participants' arm movements were entirely determined by the shape of the surfaces used.

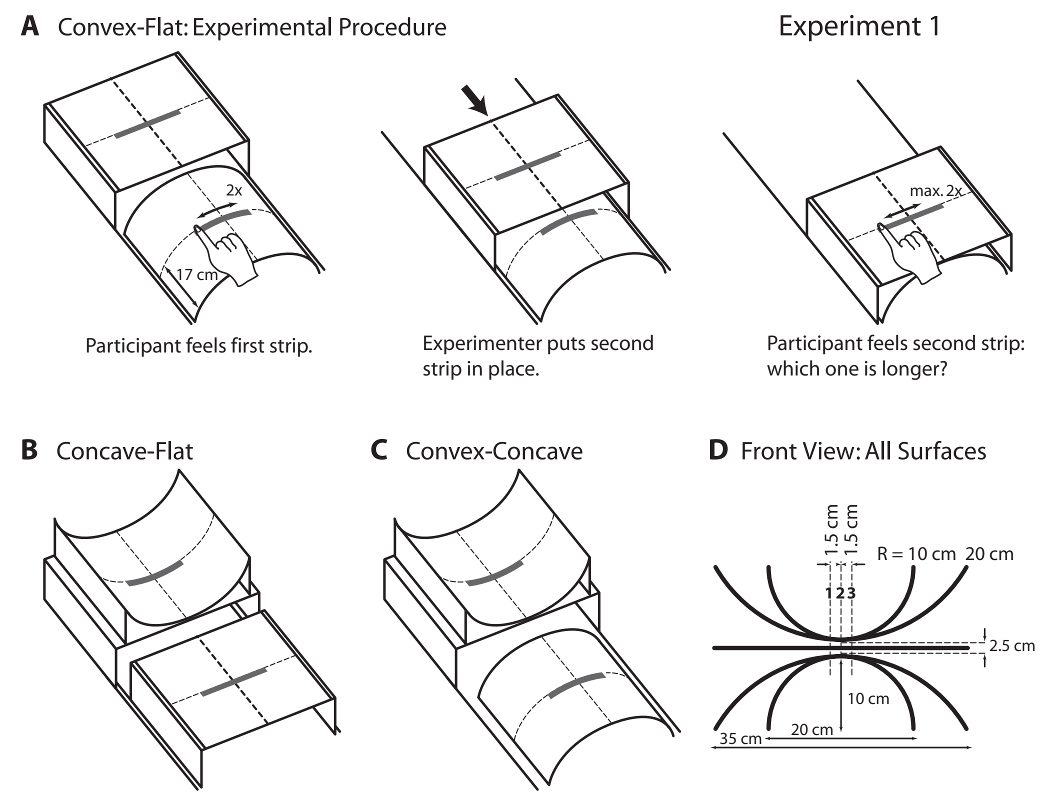

Figure 4.1. Experimental setup. The strip depicted has reference length (18 cm). Setup has orientation of Experiment 1. All three possible combinations of convex (Cx), concave (Cv), and flat (F) surfaces are shown (Panel A, B, and C, respectively). Not shown: the setup is placed on a table, behind which the observer is seated. Convex and concave are defined with respect to the finger touching the surface. Panel A illustrates the experimental procedure. Panel D shows the relative locations in space of all surfaces used.

Procedure

Measuring psychometric functions

To determine what strip lengths are perceived as equal, we measured psychometric functions according to the 2AFC method (constant stimuli). The experimental procedure is outlined in Figure 1A for the Convex-Flat condition. Participants traced the strips with the tip of the index finger of their preferred hand. A trial proceeded as follows. Participants always started at the strip's left end point. After a signal from the experimenter, they traced the first strip four times, going back and forth along the strip's edge twice. They lifted their hands and the experimenter quickly put the second strip in place. The participants then positioned their index finger at the second strip's left end point and, after a signal from the experimenter, started feeling the second strip. This strip was traced for a maximum of four sweeps. The participant then decided which of the two strips felt longer. Averaged over all observers and all conditions in Experiments 1 and 2, 6.5% of the strips was traced less than four times. The overall average number of sweeps per strip was 3.9. The time interval between the moment at which the hand was lifted from the first strip and at which it started to move along the second one was estimated to be around two seconds. The strips' positions were varied by small amounts by randomly assigning their midpoints to three possible locations (Figure 1D). If the midpoints of the strips had always been positioned at exactly the same position in space, participants could have developed a strategy for performing the task based on a comparison of the relative locations in space of either the left or the right end points. However, we wanted participants to make true distance estimates. We chose a distance of 1.5 cm between locations, because we had observed in pilot experiments that this was generally just enough for participants to notice whether or not strips were positioned symmetrically on the curved surfaces. We also informed the participants about this procedure. Thus, the participants were fully aware of the fact that information about the locations of the left or the right end points alone was not sufficient to perform this task.

The length difference between the strips on the two surfaces used in a particular condition was taken as the independent variable. It was defined as Convex - Flat, Concave - Flat, or Convex - Concave. The reference strip was 18 cm and test lengths available were 14, 16, 17, 18, 19, 20, and 22 cm. We had observed in pilot experiments that this range of test lengths was broad enough to sample the entire psychometric function from its lower tail to its upper tail and produce reliable results. A length difference Convex - Flat of, for example, +4 cm corresponded to the following four possible combinations: Cx-22, F-18; F-18, Cx-22; Cx-18, F-14; or F-14, Cx-18. Thus, the two surfaces were presented in both time orders: This was to avoid response biases. Furthermore, the reference strip was presented on both surfaces: This was to prevent learning effects, for otherwise the participants might have associated the reference length with one type of surface and might have used information from previous trials in making their judgment. For each of the seven length differences, the four possible strip combinations were repeated three times, amounting to a total of 84 2AFC trials per psychometric curve. All trials were randomized. A psychometric curve was measured in a single session, which lasted approximately between 1.5 h and 2 h. If needed, participants could take a small break halfway through the experiment. Every session started with a few practice trials (between 5 and 10). In addition, participants received a small training session of about 0.5 h prior to Experiment 1, in which they were instructed about the experimental procedure and did practice trials as well. Before the participants were seated and blindfolded, the setup was covered with a piece of cloth to prevent them from getting any visual impressions of the magnitude of the surfaces' curvatures.

Recording movement profiles

Movement profiles of the index finger were recorded by an Optotrak Certus system (Northern Digital Inc., Canada). One position marker (infrared LED) was stuck to the dorsal surface of the distal phalanx of the moving index finger. In addition, position markers were fixed to the steel plates such that we could retrieve coordinates of the magnetic strip's edge that was felt by the participant: three sensors for every curved surface, two for the flat surface. Three-dimensional spatial coordinates of the position markers were read out at a frequency of 100 Hz. Since the location of the position marker on the index finger did not correspond to the skin area touching the strip's edge, we inserted small breaks at one third and two thirds of a psychometric session, in which we asked the participants to place their index finger on the strips' end points and we gauged the finger marker's coordinates. In a single psychometric session, both end points for each of the seven test strips (midpoint location 2) and each type of surface were measured twice. By projecting the finger marker on the space curve passing through the strip's edge, we condensed three-dimensional coordinates into a single spatial parameter specifying the position of the index finger along the strip relative to the strip's midpoint.

Data Analysis

Points of Subjective Equality

We used cumulative Gaussian distributions as psychometric functions:

$$PF(x) = \tfrac{1}{2} + \tfrac{1}{2} erf\left(\tfrac{x-\mu}{\sqrt{2}\sigma}\right) \qquad (4.1),$$

where *erf* is the Error Function. The shape of this psychometric function is determined by two parameters: μ, which represents the location of the curve, and σ, which indicates its shallowness (Figure 2A). They are interpreted as follows: μ is the point at which the response score is 50% and it is assumed to represent the observer's Point of Subjective Equality (PSE), and σ is the difference between the 50% point and the 84% point and it is taken as a measure of the discrimination threshold. Psychometric curves were fitted to the data by maximizing a likelihood function, assuming that responses were generated by Bernoulli processes. Goodness-of-fit was assessed by computing log-likelihood ratios, or deviances, and comparing these against bootstrap-simulated distributions (parametric bootstrap: N = 10,000). If the experimentally obtained deviance fell between the 2.5th and 97.5th percentile of the deviance distribution, the fit was accepted. Since this was the case for all psychometric fits, from now on we will refer to parameter μ as the Point of Subjective Equality. The same bootstrap simulations were also used to estimate the variability in the PSE. Bootstrap errors are the intervals between the 16th and 84th percentile for the respective distributions and they were taken as the standard error for the PSE. For computational details we refer the reader to Wichmann and Hill (2001a, 2001b).

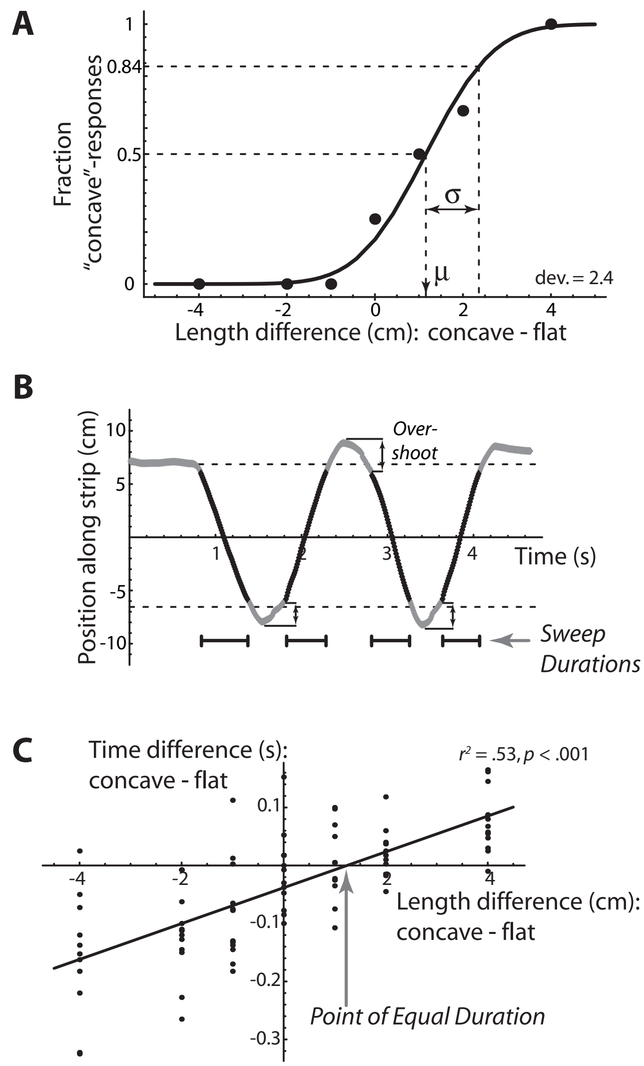

Figure 4.2. Data analysis. Panel A shows a typical psychometric curve obtained in this study. Parameter μ represents the Point of Subjective Equality. The deviance for this particular curve was 2.4. The average threshold (σ) in Experiment 1 was 1.68 cm or 9% of reference length (*SD* = 0.48 cm). Panel B shows the movement profile of the finger marker relative to the strip's midpoint for a concave strip of 14 cm. Dashed lines indicate the marker's positions corresponding to the strip's end points as recorded in the calibration measurement. One can see clearly that for the right end point the spatial coordinate of the finger marker (lower dashed line) slightly deviated from the coordinate of the end point itself (-7 cm). We extracted the times needed to traverse the inner 90% of the range between the dashed lines (black parts of the movement profile). Movement times were taken to be the mean of those sweep durations (horizontal line segments). Furthermore, note that observers generally overshot their movements along the strip (vertical arrows). Panel C shows the relation between length differences and time differences for one of the observers in condition CvF10. Time differences were defined in the same way as length differences (Convex - Flat, Concave - Flat, Convex - Concave). They were computed by subtracting the mean sweep durations as derived from the movement profiles. The x intercept of the regression line signifies the pair of strips that were traced in equal times. (All graphs are from a representative data set taken from condition CvF10, observer 6.)

Points of Equal Duration

First, we extracted those portions from the movement profiles (Figure 2B) that the finger was actually sweeping along the strip's edge (black parts). We did this by taking the inner 90% of the range between the two marker positions corresponding to the strip's end points as recorded in the calibration measurements (dashed lines). A percentage of 90% was found to be about the maximum percentage that produced reliable sweep extractions. Mean sweep durations were then used to calculate time differences between the two strips forming a psychometric trial. Time differences were defined in the same way as length differences (Convex - Flat, Concave - Flat, Convex - Concave). Next, we did a linear regression on length differences and movement time differences for every psychometric curve (Figure 2C). If the length difference is a significant predictor for the difference in sweep duration, then the x intercept of the regression line indicates the length difference between the two strips for which the times needed to trace each of them were equal. Since all regressions were highly significant (ps $< .001$), the x intercepts will hereafter be called Points of Equal Duration (PED). Error propagation was used to calculate standard errors in PEDs from errors in regression parameters.

We mention that we also did a full analysis by first doing a linear regression on position and time for each sweep and then computing sweep duration as the time needed for traversing the 90% position range according to the regression equation for that particular sweep. In addition, we did both analyses (no regression versus regression on sweeps) based on a percentage of 80% for sweep extraction. All analyses yielded the same pattern of results.

Experimental Conditions

We measured Points of Subjective Equality and Points of Equal Duration in six conditions: CxF10, CxF20, CvF10, CvF20, CxCv10, and CxCv20, representing the combinations of the convex and the flat, the concave and the flat, and the convex and the concave plates at radii of 10 and 20 cm, respectively. We used a single reference length of 18 cm in all conditions. Experimental conditions were presented in random order.

Results

PSEs and PEDs

Figure 3 shows Points of Subjective Equality for all six participants in all six experimental conditions, and Table 1 gives averages along with significance levels. All biases were negative in the Convex-Flat and Convex-Concave conditions, and they were generally positive in the Concave-Flat conditions. Condition CvF20 had a negative outlier. Consequently, average PSEs were significant in all conditions, except condition CvF20. A negative bias in the Convex-Flat conditions indicates an overestimation of the length of the convex strip relative to the flat one. For instance, a PSE of -2 cm in this condition would mean that a convex strip of 17 cm was perceived as having the same length as a flat strip of 19 cm. Similarly, positive PSEs in the Concave-Flat conditions signify underestimations of concave lengths with respect to flat lengths, and negative PSEs in the Convex-Concave conditions indicate overestimations of convex lengths relative to concave lengths.

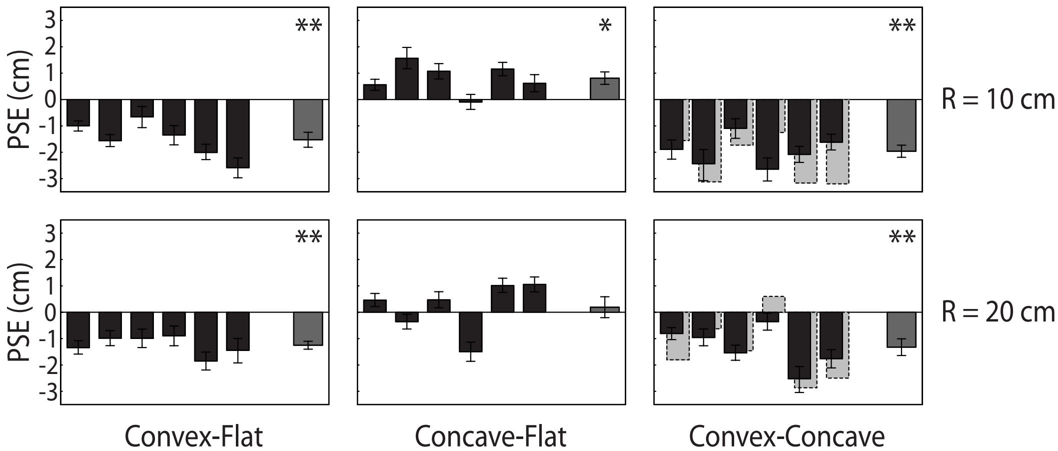

Figure 4.3. Points of Subjective Equality for all observers in each of the six experimental conditions in Experiment 1. Wide gray bars depict average PSEs for the respective conditions. These were significant in all conditions, except for CvF20. Dashed gray bars denote Convex-Concave PSEs computed from the first two conditions: Convex-Flat - Concave-Flat. Error bars for individual PSEs are bootstrap errors; error bars for mean PSEs are standard errors of the mean. $*p <$ .05. $**p < .01$.

Condition	Mean PSE (cm)	t_5	P
CxF10	-1.53**	-5.37	.003
CxF20	-1.25**	-8.36	< .001
CvF10	0.81*	3.43	.019
CvF20	0.19	0.48	.650
CxCv10	-1.96**	-8.56	< .001
CxCv20	-1.32**	-4.18	.009

Table 4.1. Mean Points of Subjective Equality in Experiment 1, $*p < .05$. $**p < .01$.

Dashed bars in Figure 3 denote Convex-Concave PSEs computed by subtracting Concave-Flat PSEs from their Convex-Flat counterparts (Convex-Concave = Convex-Flat - Concave-Flat). For condition CxCv20, a paired-samples t test showed that the difference between computed and measured PSEs was non-significant ($t_5 = 0.40$, $p = .706$), whereas the correlation coefficient between computed and measured PSEs was significant ($N = 6$, $r = .86$, $p = .027$). Therefore, for individual observers computed and measured biases were the same. For condition CxCv10, a paired-samples t test showed that the difference was also non-significant ($t_5 = 0.86$, $p = .429$), but the correlation was non-significant as well ($N = 6$, $r = .01$, $p = .989$). Note in Figure 3 that the variability in PSEs was much smaller in condition CxCv10. Thus, PSEs cluster in a narrow region, causing the correlation to be non-significant. Also for this condition we conclude that computed and measured biases agreed.

As we anticipated, magnitudes of average PSEs were larger for the smaller radius for all three combinations of surface types (Table 1). We tested whether this was a significant effect by comparing absolute values in a one-tailed paired-samples *t* test for each of the three surface combinations. However, the effect was not found to be significant (t_5 = 1.18, p = .147; t_5 = 0.10, p = .464; and t_5 = 1.36, p = .116, for CxF, CvF, and CxCv, respectively).

Figure 4 shows Points of Equal Duration together with Points of Subjective Equality, each of the six experimental conditions being represented by a different symbol. PEDs are to be interpreted in the same way as PSEs. A PED in the Convex-Flat conditions of, say, -2 cm indicates that a convex strip of 17 cm was traced in the same time as a flat strip of 19 cm. In other words, the observer needed more time to trace convex strips than flat ones of the same length. In the same manner, a positive PED in the Concave-Flat conditions signifies that the observer needed shorter times for concave strips than for flat strips, and a negative PED in the Convex-Concave conditions indicates longer times for convex strips than for concave ones. In short, PSEs and PEDs represent length differences that are subjectively equal or are traced in equal times, respectively.

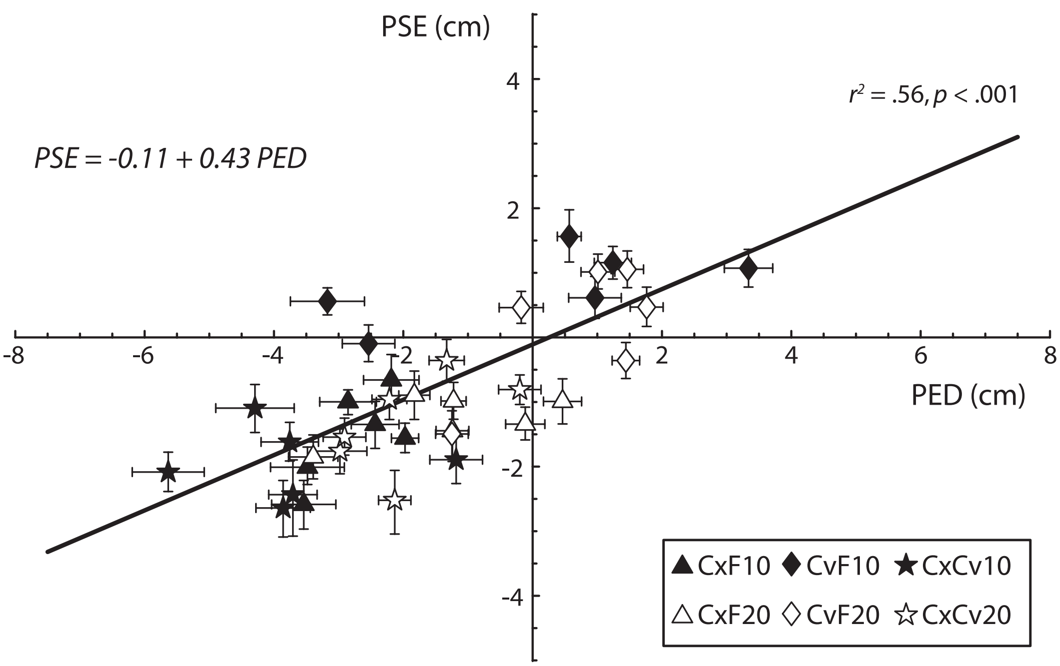

Figure 4.4. Points of Subjective Equality versus Points of Equal Duration in Experiment 1.

We did a linear regression on PSEs and PEDs, weighting data points according to their errors in both x and y (York et al., 2004). PSEs correlated significantly with PEDs (N = 36, r^2 = .56, p < .001). The best straight line corresponded to the equation $PSE = (-0.11 \pm 0.12) + (0.43 \pm 0.05)\, PED$ (standard errors indicated). Note that the participant who had a strongly negative PSE in condition CvF20 (-1.50 cm) had a similar PED (-1.25 cm). Thus, although the bias in this participant's distance estimates was opposite to the biases of the other participants, the bias closely followed his PED.

Analysis of movement duration differences

A convex strip took longer to trace than a flat one, which in turn took longer than a concave one. The question then is what caused these differences in sweep duration. At least two hypotheses can explain our results. Firstly, duration differences could simply be brought about by differences in the movement speed of the index finger. Secondly, time differences could have been caused by differences in what we will call contact points for the left and right corner of the strip. We define the contact point as the skin area where the observer feels the strip's end point. If the contact points for the left and right end point of the strip are located on different parts of the finger (Figure 5A, left part), then the traced length will be shortened or lengthened by the distance between the contact points (Figure 5A, right part). Note that this hypothesis does not explain the cause of directional shifts in contact points. It tells us only that there is a shift in contact point which is opposite for convex and concave surfaces, and we assume that this shift is symmetric. Shifts as indicated in Figure 5A would agree with differences in movement durations as found in Experiment 1.

At this point, it is important to realize that we cannot find evidence in favor of or against the contact point hypothesis by observing raw movement profiles, as for example in Figure 2B. For this particular movement profile, the spatial coordinate of the right end point (-7 cm) deviated slightly from the corresponding coordinate of the position marker as recorded in the calibration experiment (lower dashed line). However, remember that we recorded the movement of the index finger by reading out coordinates of a position marker that was stuck to an arbitrary location on the dorsal surface of the distal phalanx. So, the location of this position marker did not correspond to the skin area that was actually touching the strip's edge. Thus, unfortunately the data do not allow us to deduce from the coordinates of this position marker whether contact points were indeed different, since differences in the location of the position marker relative to the left and right strip's end point could have been provoked not only by differences in contact point but also by rotations of the index finger relative to the strip which keep the contact point the same. From the raw movement profiles we can only extract the times that the participants associated with movement along the strip.

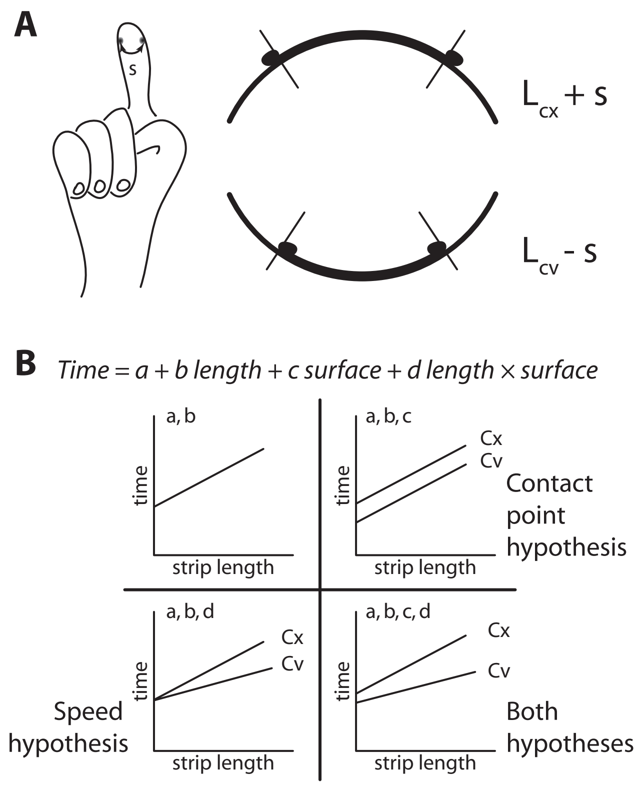

Figure 4.5. Analysis of movement duration differences. Panel A illustrates the contact points hypothesis. According to this hypothesis differences in sweep durations are caused by a shift *s* in the contact points for the left and right end points of the strip. The traced distance gets lengthened or shortened by the amount of the shift for convex or concave surfaces, respectively. Panel B explains the multiple regression model for sweep durations. The contact point hypothesis predicts that parameter *c* is significant, and the speed hypothesis predicts that parameter *d* makes a significant contribution to the model.

However, the two hypotheses can be tested against each other by considering the relationship between movement times and actual strip length they predict. For instance, in the case of the Convex-Concave conditions, the speed hypothesis predicts

$$T_{Cx} = L_{Cx} / v_{Cx} \quad \text{versus} \quad T_{Cv} = L_{Cv} / v_{Cv} \qquad (4.2),$$

where T, L, and v denote movement time, actual strip length and average speed for the respective surface types. The contact point hypothesis, on the other hand, predicts

$$T_{Cx} = (L_{Cx} + s) / v \quad \text{versus} \quad T_{Cv} = (L_{Cv} - s) / v \qquad (4.3),$$

in which s signifies the shift in contact point (Figure 5A) and v the average speed, which in this case is the same for both surfaces. Thus, the speed hypothesis predicts a difference in slope, whereas the contact point hypothesis predicts different constant terms.

Therefore, we did a multiple regression on each of the six experimental conditions, combining the data for all participants (N = 6 observers x 2 surface types x 84 trials). As a measure of movement time, we took the mean sweep duration (Figure 2B). The predictors we considered were Length, Surface Type, and their interaction Length x Surface Type (Figure 5B). The regression equation is *Time* = *a* + *b Length* + *c Surface Type* + *d Length* x *Surface Type*. The predictor Length was always included in the model. We did a forward regression for Surface Type and Length x Surface Type: After Length had been included, the predictor that accounted for the largest portion of the remaining unexplained variance in movement times (largest semi-partial correlation) was added and retained, if it made a significant improvement to the model. The predictor that remained was considered likewise. If only the Surface term is added to the model, this would mean that there is a significant difference in constant terms for the two surface types, which supports the contact point hypothesis. If the interaction term is included in the model, there is a significant slope difference, which shows that the movement speeds across the surfaces were different. Finally, if the Surface term as well as the interaction term are included, then differences in sweep durations are due to both speed differences and different contact points.

Table 2 gives the results of the multiple regression analysis. Apart from the Length term, the regression model included the interaction term but not the Surface term in five out of six conditions, showing that in these conditions different movement speeds provide the best prediction of the differences in movement times for the various surface combinations. In one condition, only the Length term constituted the regression model. Thus, the contact point hypothesis is rejected in favor of the speed hypothesis. Table 2 also gives relative speed differences as computed from the regression parameters.

	Regression model					Speed
Condition	*a* (10^{-2} s)	*b* (10^{-2} s/cm)	*c* (10^{-2} s)	*d* (10^{-2} s/cm)	R	difference
CxF10	11.31	6.94	-	-1.02	.45	-14.6%
CxF20	0.19	6.61	-	-0.39	.29	-5.9%
CvF10	10.05	5.63	-	-	.27	-
CvF20	19.31	4.71	-	0.20	.32	0.7%
CxCv10	2.51	7.30	-	-1.40	.37	-29.4%
CxCv20	10.01	5.66	-	-0.66	.41	-11.7%

Table 4.2. Multiple regression analysis of sweep durations (N = 1,008), and relative speed differences in Experiment 1. Regression equation: *Time* = *a* + *b Length* + *c Surface Type* + *d Length* x *Surface Type*. Surface Type term: convex = 0 and flat = 1 for Convex-Flat conditions, concave = 0 and flat = 1 for Concave-Flat conditions, convex = 0 and concave = 1 for Convex-Concave conditions. Dashes indicate that the predictor's contribution to the model was non-significant or the relative speed difference could not be computed. All models: $p < .001$. Relative speed differences between surface types are computed from regression parameters *b* and *d*. Speed differences are defined as (Cx - F)/F, (Cv - F)/F and (Cx - Cv)/Cv for the Convex-Flat, Concave-Flat and Convex-Concave conditions, respectively.

Other variables

We investigated whether the magnitude of the PSE depended on the overall average movement speed of the index finger. First, we computed the average speed per condition for each participant (i.e., the average speed applied in a single psychometric session) from the slope of the straight line fitted to mean sweep durations versus strip lengths for the aggregated data of the two surface types. Note that this is an average of movement speeds across the two surfaces tested in that particular condition. Movement speeds could differ by as much as a factor of three (*Min* = 9.8 cm/s, *Max* = 29.3 cm/s, *Mdn* = 15.7 cm/s, *IQR* = 5.0 cm/s). There was no significant correlation between PSE magnitudes and movement speeds ($N = 36$, $r = -.19$, $p = .270$).

Furthermore, we tested whether there were systematic differences in overshooting movements between the various surface types. To exclude any effect of strip length, we only analyzed trials in which both strips were 18 cm ($N = 6$ observers x 12 trials). First, we computed the mean extent of the overshooting movement for each movement profile (Figure 2B). In case of four sweeps, there were three overshooting movements. Occasionally, a trial had to be excluded from analysis, because only a single sweep was made and consequently no overshooting movement could reliably be defined. The overall average extent of the overshooting movement at reference length was 17 mm. Next, we performed a paired-samples *t* test for each of the six experimental conditions. The difference in overshooting extent for the two surfaces used in a particular condition was defined in the same way as the length difference for that condition (Convex - Flat, Concave - Flat, or Convex - Concave). For three conditions, the difference in overshooting extent was significant and in agreement with the direction of the corresponding PSE ($M = -2.9$ mm, $t_{71} = -7.34$, $p < .001$; $M = -0.9$ mm, $t_{70} = -2.88$, $p = .005$; and $M = 0.9$ mm, $t_{71} = 2.35$, $p = .022$, for CvF10, CvF20, and CxCv20, respectively). For example, in case of condition CvF10, overshooting was smaller for the concave strip by about 3 mm, while at the same time concave lengths were underestimated. However, for two conditions the difference in overshooting extent was significant but opposite to the direction of the PSE for that condition ($M = -1.4$ mm, $t_{69} = -3.39$, $p = .001$, for CxF10; and $M = -1.4$ mm, $t_{69} = -2.77$, $p = .007$, for CxCv10). For condition CxF20, the difference was non-significant ($M = -0.03$ mm, $t_{68} = -0.09$, $p = .927$).

For reasons that will be explained in the Discussion, we also computed instantaneous movement speeds at the very start and end of a sweep by taking the three-point derivative (Press et al., 2002). We assume that the starting and ending speeds of a sweep are indicative of the speeds at which the index finger passes the first and last end points of a strip. Again, we only analyzed trials in which both strips had reference length. For each movement profile, we then averaged starting and ending speeds over all sweeps. The starting speed for the first sweep was excluded from analysis. We were interested in the difference between the mean starting and ending speed and whether this difference varied systematically between the various surface types. Paired-samples *t* tests showed that the effect was significant only in condition CxF10 ($M = -0.95$ cm/s, $t_{69} = 2.57$, $p = .012$). For both the convex and flat surface in this condition, starting speeds were on average higher than ending speeds, with the difference between starting and ending speed being in turn 0.95 cm/s larger on the flat surface compared to the convex one. For all other conditions, differences were non-significant (all ps > .183).

Discussion

We found that convex and concave surfaces had qualitatively different effects on observers' length percepts: Convex lengths were overestimated, whereas concave lengths were underestimated compared to flat strips. Thus, not only whether a surface was curved or flat but also the sign of curvature affected observers' length estimates and any explanation for curvature biases would have to account for this qualitative difference.

If a convex length is overestimated relative to a flat one, and a concave length is underestimated relative to a flat length, then in turn, one would expect convex lengths to be overestimated relative to concave lengths. The results did indeed indicate that measured Convex-Concave PSEs and Convex-Concave PSEs computed from the first two conditions agreed very well. Not only does this finding demonstrate that the experiment is highly reproducible, but it also shows that the experimental conditions are consistent in the sense that observers appear to use the same mechanism in each condition to draw inferences about the length of the traced edge.

If curvature influences an observer's perception of length, then one would also expect to find smaller PSEs for smaller curvatures, or larger radii, since in the limit case the observer would essentially be comparing the lengths of flat strips and, given the experimental design, we do not anticipate any biases in that case. Although there was a trend in the data for PSEs to be smaller for the larger radius, this trend was not significant. Apparently, the difference between the 10-cm radius and the 20-cm radius was too small for any effect on PSEs to be detected.

As a next step in our analysis, we showed that there was a positive and significant correlation between Points of Subjective Equality and Points of Equal Duration. We were surprised by the considerable correlation between subjective estimates and movement characteristics, given that we are investigating conditions of relatively unconstrained exploration. Thus, observers' length estimates followed durations of movement and, taken together with previous research on haptic length perception, this suggests that observers indeed took movement time as a measure of perceived length. We also demonstrated that differences in movement duration derived from differences in movement speeds. The index finger moved slowest along the convex surface and fastest along the concave one, which led to longer movement times for convex lengths.

Cause of speed differences

How can we explain these speed differences? Why did the observers move their index finger at different speeds across the different surface types? Several hypotheses can be put forward to account for this effect. Our first hypothesis relates speed differences to differences in moments of inertia for convex and concave movements. This explanation is the same as the inertial anisotropy hypothesis proposed for the RT effect. We designed the setup such that postures of the observer's arm were as similar as possible for all surface types, but, of course, movements across different surfaces require different combinations of joint rotations. In the course of the experiments, we observed that participants made more whole-arm movements and had less wrist rotation for convex surfaces, whereas for concave movements the opposite was the case. Thus, inertial resistance might be larger for convex movements than for concave movements and so the former would require more energy. If the observer does not compensate for this inertial anisotropy, the resulting movement speed would indeed be lower for the convex surface than for the concave surface, as we found in our experiment.

The second hypothesis has been suggested by one of the participants. This participant noticed that while the finger moves across the surfaces, it can more easily lose contact with a convex surface than with a concave one. Indeed, if at some point during movement the observer stops exerting forces, the index finger will in rough approximation continue to move along a straight line tangential to the surface at that particular point. For the concave surface, the index finger is then pressed against the steel plate, whereas for the convex surface the index finger, so to speak, takes off. One could say that the finger moves on the inside of the plate for the concave surface and on the outside for the convex surface. To prevent this from happening, the observers will move their index finger slightly more slowly across the convex surface.

If the take-off hypothesis were indeed to explain differences in movement speeds and if observers did not take these speed differences into account, then one might expect PSEs to be larger for observers who move their index finger at higher speeds, as it is easier to lose contact with the convex surface when moving faster. However, we did not find any correlation between PSE magnitudes and movement speeds. This argues against the take-off hypothesis, but does not disprove it, since it could still very well be that observers who move their index finger at higher speeds simply press their finger with a greater force against the surface, thereby canceling the effect of movement speed.

A third hypothesis is that gravity modulates movement speeds along the different surface types. Observers try to move at equal speeds across all surfaces, but they do not succeed because they do not fully compensate for gravity effects. It is not yet clear precisely in what way this effect is brought about, but we can reasonably assume that gravity acts differently on convex and concave movements. For example, when observers start moving their index finger along a convex edge, they accelerate in a direction (upwards and to the left or right) that is partially opposite to gravity (downwards). Towards the end of the sweep they decelerate and in that case gravity is again working against their movement. To put it more precisely, for convex surfaces the component of the gravity vector that is tangential to the surface opposes the acceleration and deceleration phases of the movement (Figure 6). For concave surfaces, on the other hand, the opposite is the case. Note that a possible gravity effect on movement speeds would be similar to the effect that the added resistive force had in one of the active conditions in McFarland and Soechting's (2007) research. The absence of any effect of this manipulation on the RT illusion argues against the gravity hypothesis.

Secondary effects

We will now briefly discuss two secondary effects that may possibly have influenced length estimations in addition to the effects of movement speed as discussed above. We will show that effects are negligible or can even be discarded.

First, there is a possibility of secondary effects of overshooting on length estimations. Additional biases in length judgments could have been introduced due to overshooting movements being larger for one or the other surface type. However, differences in overshooting extent were much smaller than actual PSEs reported. Moreover, this effect, if any, must interact with the effect of movement speed, since only for three out of six conditions did differences in overshooting extent agree with the direction of the PSE.

Second, Dassonville (1995) reports that observers who receive a short tactual vibration to the index finger while making a fast pointing movement to a visual target, mislocalize the location of this vibration at a position that lies a fixed time interval beyond the actual location of the tactual stimulus. In case of length estimations, the locations of the end points of the strip may thus be misperceived, affecting perceived length in turn. Suppose that the perceived location of the first end point that is passed in a sweep is shifted along the curved trajectory in the direction of movement: The perceived location lies somewhere on the strip itself. The location of the last end point is shifted in a similar fashion, and it is now perceived to lie somewhere beyond the actual strip's end point. Since the shifts are assumed to be constant when expressed in time, the shifts expressed in distance depend on the velocities of the moving index finger at the moment that the end points were crossed. If the velocity at which the first end point in a sweep is crossed is larger than the velocity at which the last end point is crossed, a small shrinkage of perceived length would occur: The shrinkage equals the difference between velocities at the first and last end points multiplied by the time constant. Since in our experiment observers had to discriminate lengths across different surfaces, secondary effects may occur if speed differences between first and last end points vary systematically between the various surface types. As we have shown, in only one case were speed differences between first and last end points systematically different for the two surface types tested, but the direction of this effect was opposite to the direction of the reported length biases.

Experiment 2

In Experiment 2, we tested the gravity hypothesis by having participants touch convex and concave surfaces from below instead of from above, as they did in Experiment 1. Recall that we defined convex and concave with respect to the index finger touching the surface. Thus, a particular surface that is convex when it is touched from above in Experiment 1 will be called concave when the same surface is touched from below in Experiment 2. Because movement directions of the index finger relative to the tangential component of the gravity vector are then the same for a convex surface in Experiment 1 and a concave surface in Experiment 2 (Figure 6), we hypothesized that gravity effects would be alike for these two surfaces. Likewise, we predicted that gravity would modulate movement speeds similarly for a concave surface in Experiment 1 and a convex surface in Experiment 2.

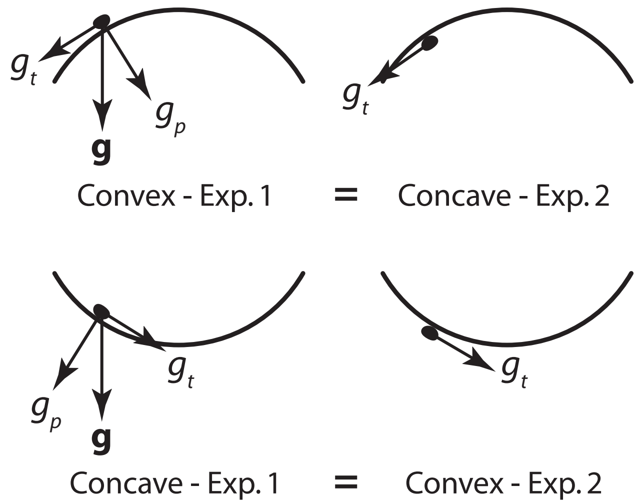

Figure 4.6. The gravity vector (**g**) acting on the moving index finger can be decomposed into a component tangential to the surface (g_t) and a perpendicular component (g_p). The gravity hypothesis predicts that the effect of the tangential component on speed profiles of the moving index finger is the same for convex surfaces in Experiment 1 and concave surfaces in Experiment 2, and vice versa.

Thus, we expected PEDs to reverse sign according to the gravity hypothesis, and in addition, if participants still took duration of movement as a measure of perceived length, we expected PSEs to reverse sign as well. On the other hand, both the inertial anisotropy and take-off hypothesis would predict that PEDs and consequently PSEs do not reverse sign, since curvature biases explained by these two hypotheses are linked to the sign of curvature.

To rule out any constant effects of reversing the setup, we will only be comparing movement times for combinations of surfaces having the same orientation. We made postures of the right arm in Experiment 2 as similar as possible to postures in Experiment 1 by positioning and orienting the curved strips with respect to the participant exactly the same in both experiments. The only difference between the two experiments was that the participant had to rotate the forearm by a half turn to be able to touch the strips. Thus, in the following experiment we assumed that having the palm of the hand facing up (supination) will at most introduce a constant effect for all surfaces types compared to having the palm of the hand facing down (pronation, Experiment 1).

Method

The same observers participated in Experiment 2. They finished Experiment 1 before starting Experiment 2. The setup from Experiment 1 was now oriented upside down, and the surfaces were touched from below (Figure 7A). Participants assumed the same position with respect to the setup: The setup's midline had exactly the same location and orientation relative to the participants as in Experiment 1. The range in which the hand can be rotated is narrower when the fore arm is in supination than in pronation, especially for movement of the index finger towards the right end point of the strip. We decided to test the 20-cm radius, because we found that for this radius every participant could still comfortably reach both strip end points and beyond with the index finger, even for the largest test length (22 cm). We measured Points of Subjective Equality and Points of Equal Duration in the same way as in Experiment 1. All three combinations of surface types were tested: CxF20, CvF20, and CxCv20 (Figure 7B). The procedure was the same as in Experiment 1.

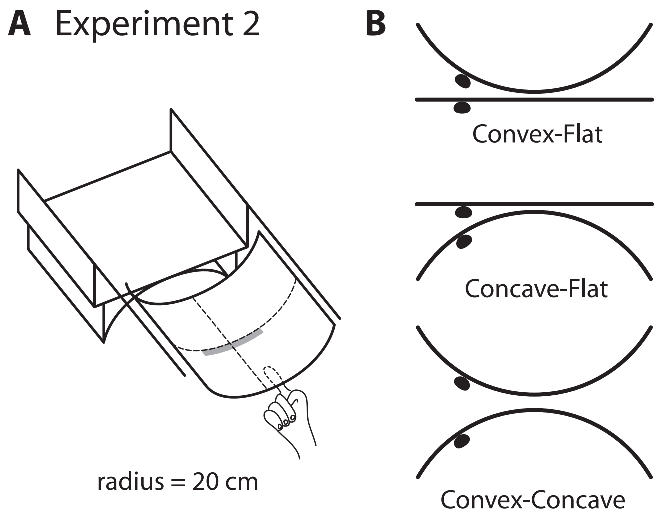

Figure 4.7. Setup and experimental conditions in Experiment 2. (A) The setup was identical to the one used in Experiment 1, except that it was now oriented upside down. Steel plates were touched from below. (B) All three combinations of the convex, concave, and flat plates were tested at a single radius of 20 cm.

Results

Figure 8 shows the results for each of the three experimental conditions. Since Experiment 2 was designed to test the gravity hypothesis, which makes predictions only about movement speeds, each bar chart now shows PEDs first and PSEs second. We will discuss the results for each of the three experimental conditions separately, because there were qualitative differences between the conditions.

There was an appreciable correlation between movement durations and length estimates for the Convex-Flat condition, whereas the correlation is lost for the Concave-Flat and Convex-Concave conditions. Correlations are now based on only six data points and none of them reached significance. However, the important observation here is that the correlation for the Convex-Flat condition is considerably higher than for the other two conditions. In other words, as soon as the Concave surface comes into play, the correlation between movement durations and subjective length estimates drops to near zero.

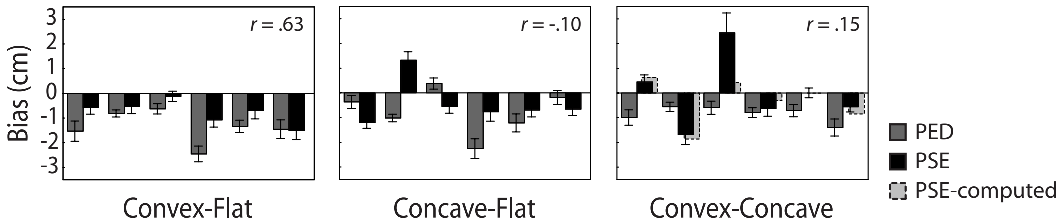

Figure 4.8. Points of Equal Duration and Points of Subjective Equality for each condition in Experiment 2. None of the correlations (N = 6) was significant (p = .178, p = .856, and p = .771, for CxF20, CvF20, and CxCv20, respectively). Dashed gray bars indicate Convex-Concave biases computed from the first two conditions: Convex-Flat - Concave-Flat.

As in Experiment 1, we tested whether differences in movement durations were indeed caused by differences in movement speeds. The results of the multiple regression analysis are given in Table 3. In all three conditions, the regression analysis supports the speed hypothesis but not the contact point hypothesis. Table 3 also gives relative speed differences as computed from the regression parameters. In Experiment 1, the participants moved their finger slowest across the convex surface and fastest across the concave one, the flat surface being traversed at intermediate speed. In Experiment 2, the convex strips are again traced at lowest speed and the flat strips at highest speed; this time it was the concave strips that were traced at intermediate speed.

Condition	Regression model					Speed difference
	a (10^{-2} s)	b (10^{-2} s/cm)	c (10^{-2} s)	d (10^{-2} s/cm)	R	
CxF20	6.12	3.91	-	-0.29	.34	-7.3%
CvF20	4.88	3.81	-	-0.13	.34	-3.4%
CxCv20	7.46	4.12	-	-0.20	.31	-4.9%

Table 4.3. Multiple regression analysis of sweep durations (N = 1,008), and relative speed differences in Experiment 2. Regression equation: *Time* = *a* + *b Length* + *c Surface Type* + *d Length* x *Surface Type*. Surface Type term: convex = 0 and flat = 1 for the Convex-Flat condition, concave = 0 and flat = 1 for the Concave-Flat condition, convex = 0 and concave = 1 for the Convex-Concave condition. Dashes indicate that the predictor's contribution to the model was non-significant. All models: $p < .001$. Relative speed differences between surface types are computed from regression parameters b and d. Speed differences are defined as (Cx - F)/F, (Cv - F)/F and (Cx - Cv)/Cv for the Convex-Flat, Concave-Flat and Convex-Concave condition, respectively.

Next, Table 4 gives average PSEs for the three experimental conditions together with significance levels. Only in condition CxF20 was there a significant negative bias. Therefore, this condition replicates condition CxF20 in Experiment 1: First of all, participants systematically moved their finger more slowly across the convex surface than across the flat surface, as in Experiment 1; and second, their length estimates followed the duration of movement and consequently they overestimated the convex length compared to flat, again as in Experiment 1.

Condition	Mean PSE (cm)	t_5	p
CxF20	-0.75*	-3.84	.012
CvF20	-0.42	-1.17	.295
CxCv20	0.00	0.00	.998

Table 4.4. Mean Points of Subjective Equality in Experiment 2. *p < .05.

As in Experiment 1, we analyzed overshooting movements for trials in which both strips had reference length. In all three conditions, extents of overshooting were significantly different for the respective surface combinations, but differences were again very small (M = -1.3, t_{71} = -2.95, p = .004; M = -3.0, t_{71} = -6.50, p < .001; and M = 1.40, t_{71} = 3.35, p = .001, for conditions CxF20, CvF20, and CxCv20, respectively). In case of the first two conditions, for which nonzero PSEs were found, the difference in overshooting extent was opposite to the direction of the reported bias.

Finally, we did not observe learning effects. There was no significant difference in thresholds between Experiments 1 and 2 as shown by a paired-samples t test on average thresholds per observer (t_5 = 0.72, p = .503).

Discussion

In Experiment 2, we set out to investigate whether the speed differences we found in Experiment 1 resulted from gravity effects. We argued that if this were the case, PEDs and consequently PSEs should reverse sign when the surfaces used in Experiment 1 are now touched from below. The results for condition CxF20 disproved the gravity hypothesis.

In the other two conditions, CvF20 and CxCv20, there was no longer any overall correlation between movement durations and length judgments. However, the strong agreement between measured and computed PSEs for individual observers indicates that the Convex-Concave condition is consistent with the Convex-Flat and Concave-Flat conditions. Thus, we think that by reversing the setup we have introduced an additional curvature bias for the Concave surface. We can only speculate as to the origin of this additional curvature bias. We do know, however, that first of all this bias is not connected to the duration of movement, and second, that its effect is highly idiosyncratic, because the variability in PSEs was much higher in the conditions that involved the concave surface.

General Discussion

In this research, we have demonstrated that curvature induces systematic errors in haptic length perception. Observers made active movements to trace a curved pathway and judge the length of it. Surprisingly, we found in Experiment 1 that the sign of curvature had a differential effect on length estimates: convex lengths were overestimated, whereas concave lengths were underestimated. We have also shown that a kinematic mechanism underlies length estimates: Curvature biases probably originate from a discrepancy between actual kinematic properties of arm movements and kinematic properties as inferred by the observer.

In particular, we observed a significant correlation between movement times and length estimates. The differences in movement durations for convex and concave lengths were caused by differences in movement speeds across the different surface types. For example, the index finger moved slowest across the convex surface, which led to longer movement times for this type of surface. Two questions then arise: First, what induces these speed differences, and second, what is the basis for the observer's misjudgments of curved lengths?

One could come up with many complex explanations to account for systematic errors in length estimates when observers use limb movements to judge distance traversed. Note that any explanation involves taking a number of different steps. Since distance traversed is inferred from the kinematics of the tracing movement, that is to say, from the characteristics of the movement profile, one would have to explain not only what brings about particular movement characteristics. One would have to explain also how the observer perceives properties such as time, velocity, acceleration, etc. for movements along different pathways and in what particular way these percepts are processed to constitute a length percept. We now discuss two types of mechanisms that fit with our data set. Each mechanism has to answer the two questions stated above. According to the first account, curvature effects are due to a discrepancy between planned or desired movement profiles on the one hand and profiles as they are actually realized on the other hand, and, as such, the curvature effects have the nature of a motor error. According to the second mechanism, systematic biases in the perception of movement speeds for convex and concave surfaces cause the observers to move their finger at different speeds across the two surfaces while perceiving their movement speeds to be the same: This mechanism explains the curvature effects in terms of a perceptual error.

Undetected Speed Differences

According to the first explanation, observers plan to move their finger with equal --physical-- speeds across the different surface types, in which case movement duration would be a veridical measure of distance traversed. However, motor commands sent to the muscles do not meet this requirement. For example, observers do not fully compensate for inertial anisotropies of convex and concave arm movements or for effects of gravity, and consequently in neither case do they satisfy the dynamic constraints of the desired arm movements. These factors can be called curvature-extrinsic, because they are not specific to curvature but would arise in different settings too: The inertial anisotropy hypothesis has been proposed to explain the RT effect (Wong, 1977) and one could imagine that gravity effects also play a role in tasks that differ from ours. The illusion must then arise because observers think they are moving their finger at the same speed across the different surface types so that they can take time as a measure of perceived length.

An important question that has to be answered is why observers do not notice speed differences. What does it actually mean if speed differences are undetected? This question has not been addressed in earlier research. One might conclude that speed differences have been too small for the observer to detect. However, these small speed differences lead to differences in movement duration, which in turn yield robust curvature biases. Although mechanisms for perceiving time and movement speed of limbs may be quite distinct, it is reasonable to assume that these mechanisms are tuned to each other. Thus, we think it is unlikely that sub-threshold speed differences would lead to supra-threshold time differences. Another possibility is that sensory feedback indicating differences in actual movement speeds is simply ignored due to the cognitive load of the task, and observers assume from movement onset that they are moving their index finger at the same speed. Because participants generally make four sweeps and can choose their own speed range, this possibility also seems unlikely.

Misperceived Speeds

A second explanation that could account for the reported curvature effects is that perception of speed is not veridical for convex and concave surfaces. In other words, speeds across the two surface types are perceived differently, which in turn causes curvature biases in length perception. Note that when we are talking about movement speeds of the index finger, we are actually referring to speeds in external space, that is, without any reference to the body (in this case: speed with respect to the strip's edge). Given then that the sense of speed of limb movements is the result of complex transformations starting from signals from muscle spindles, mechanoreceptors, and possibly joint receptors, and given that physically simple trajectories are generally complex in biomechanical terms (e.g., movement of finger along a straight line), distortions in the spatial representations that are a prerequisite for these transformations might account for the misperception of speeds across convex and concave surfaces.

The similar results for the active and passive RT effect (McFarland and Soechting, 2007) lend support to this explanation, considering that at a physiological level there are considerable differences between active and passive arm movements. For example, there is no motor outflow for passive movements, and consequently no corollary discharges are generated to anticipate the reafferent sensory effects of movements (Miall and Wolpert, 1996). Furthermore, sensitivity of muscle spindles is severely reduced for passive movements, because due to the absence of motor commands there is no alpha-gamma co-activation (Clark and Horch, 1986). Finally, transmission of cutaneous feedback is decreased during voluntary movements (e.g., Chapman, 1994; Seki et al., 2003), and detection thresholds for tactile stimuli are correspondingly increased, the increase appearing more intense for active movements than for passive movements (e.g., Chapman and Beauchamp, 2006;

Vitello et al., 2006). Tactile sensitivity at the contact between finger and surface plays an important role in haptic length perception, because cutaneous feedback has to be monitored during active movement to check whether the limb is still in contact with the stimulus and because tactile slip may be an important cue for movement speed (Dépeault et al., 2008; Essick et al., 1988).

An example of such a spatial distortion accounting for the misperception of movement speeds might be the radius of curvature being perceived differently for convex and concave surfaces. Curvature information is probably a crucial type of spatial knowledge needed for computing movement speeds. Suppose that the convex radius is overestimated by an amount δ, whereas the concave radius is underestimated by the same amount (Figure 9). Then, the pathway that the observer perceives to be tracing (thin line) is different from the actual surface (thick line): The perceived convex trajectory is stretched and the perceived concave trajectory is shrunk with respect to the real surface. Therefore, if the observers move their finger at equal speeds along these perceptual trajectories, physical movement speeds (i.e., the speeds that we actually measured) are lower for the convex than for the concave surface, as we found in Experiment 1. Let us assume that the observers' finger did indeed maintain equal speeds along the perceptual trajectories, then take

$$PSE = L_{Cx}^{PSE} - L_{Cv}^{PSE} \quad (4.4),$$

and since

$$L_{Cx}^{PSE} + L_{Cv}^{PSE} = 36cm \quad (4.5),$$

we can solve for δ in

$$L_{Cx}^{PSE} \frac{R+\delta}{R} = L_{Cv}^{PSE} \frac{R-\delta}{R} \qquad (4.6),$$

in which R is the actual radius and L^{PSE} the length of the pathway for the respective surface at PSE. Table 5 gives values for δ based on average PSEs for each of the six experimental conditions in Experiment 1 as reported in Table 1.

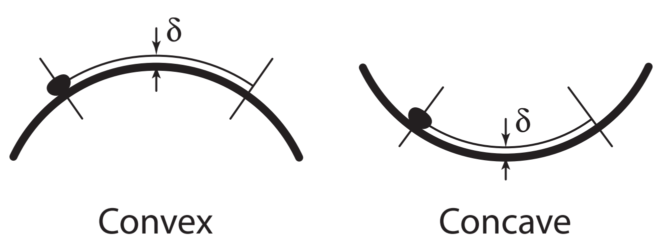

Figure 4.9. Misperceived radii of curvature for convex and concave surfaces. Convex radius is overestimated by an amount δ and concave radius is underestimated by the same amount. Observer maintains equal speeds along perceptual trajectories (thin lines), causing speed differences along actual surfaces (thick lines).

Condition	δ (cm)
CxF10	0.89
CxF20	1.44
CvF10	0.44
CvF20	0.21
CxCv10	0.54
CxCv20	0.74

Table 4.5. Overestimations of convex and underestimations of concave radii, based on model for misjudgments of curved lengths in Experiment 1. Calculations are based on average PSEs for the respective conditions.

Previous research on haptic curvature discrimination (notably Pont et al., 1999) addressed the question of the possible cues conveying curvature information (height differences, attitude differences, local contact curvature information). Interestingly, Pont et al. (1999) showed that scanning length affects one's curvature percept, whereas we have shown that curvature affects one's length percept. However, in all their experiments participants had to take the sign of curvature into account. Van der Horst and Kappers (in press) did discrimination experiments in which observers had to decide which of two curvatures, one being convex and the other being concave, felt more curved. Observers thus had to disregard the sign of curvature. Surprisingly, for stimuli that had dimensions, orientations and curvatures similar to the surfaces used in this study, they found a systematic bias towards overestimations of radii of convex curvatures compared to concave. In other words, a particular curvature appeared flatter when it was convex than when it was concave. Unfortunately, Van der Horst and Kappers (in press) did not control for scanning length, which makes it hard to compare their results with those of the present study. The point that we want to make here is that the systematic overestimations of convex and underestimations of concave lengths as reported in this paper might be caused by an interaction between length and curvature percepts, tentatively having the nature of a percept-percept coupling (Epstein, 1982). Finally, we find it intriguing that typical values for δ range roughly between 0.5 cm and 1 cm, which is the same range as the thickness of the index finger. We might then hypothesize that the part of the finger that the observers perceive to be tracing the pathway is not the skin area contacting the strip's edge, as it should have been, but is some point located more centrally inside the index finger.

Other Mechanisms?

In discussing possible mechanisms underlying overestimations of convex and underestimations of concave lengths, we have provided concrete explanations for the occurrence of these curvature biases. As we already remarked, one can come up with other explanations: There may be mechanisms that work at a more cognitive level. For instance, perception of duration might be different for different types of movements, because certain movements are more complex and therefore require more cognitive effort (e.g., Sadalla and Magel, 1980; cf. Proffitt et al., 2003). However, because cognitive effort is at present ill-defined, it is not clear a priori why one or the other type of movement would involve more effort. This explanation is therefore difficult to test.

RT Effect

In this study, we have presented a new experiment in which observers make limb movements in order to judge distance traversed. As we argued in the Introduction, it is reasonable to assume that the RT effect and the curvature effects reported in this paper share a common origin. Thus, by comparing these different experimental tasks we may transcend task dependencies and come closer to understanding the general principles governing distance estimates. For example, if it can indeed be shown that a distorted perception of convex and concave radii causes curvature effects, the interesting question that arises is whether an equivalent spatial distortion underlies the RT effect. Furthermore, McFarland and Soechting's (2007) research predicts that if the index finger is being moved passively across convex and concave surfaces at equal speeds, convex lengths are still overestimated and concave lengths underestimated. Thus, their research on the RT effect argues against the undetected speed differences explanation and in favor of the misperceived speeds mechanism for curvature biases in this report.

Conclusions

In this paper, we have shown that curvature affects haptic length perception. Results of Experiment 2 provide indications that biases in Experiment 1 are not due to gravity effects. Taking into account research on the RT effect as discussed in the Introduction and research on curvature perception as discussed above, we might guess that the basis for the illusion is a spatial distortion that causes movement speeds to be misperceived for convex and concave surfaces. Further research is needed to reveal the mechanisms underlying the reported effects.

Interestingly, two studies (Norman et al., 2000; Norman et al., 2004) reported analogous length biases in visual perception. Participants estimated lengths across flat and curved surfaces (cylinders, convex and concave hemispheres, saddle shapes). These surfaces were real textured objects and were viewed stereoscopically: a full cue situation that might be comparable to the relatively unconstrained manner of exploration in the present study. Estimation of lengths across curved surfaces was distorted also in the visual case. Particularly, lengths along the curved dimension of a convex cylinder were generally overestimated, although there was a dependency on the cylinder's orientation and distance to the cylinder (Norman et al., 2000). Apparently, perception of length across curved surfaces is distorted, no matter whether the surfaces are defined visually or haptically.

An interesting implication of the reported curvature effects is that local shape apparently interacts with length perception. This means that distance estimates are not informative unless the geometry of the path is specified. Since movement of the hand or fingers across the surface of an object is a common exploration procedure for perceiving an object's shape (Klatzky and Lederman, 2003), the question is whether or how the effects reported here influence perception of an object's overall shape. Perhaps distance information is used for making the transition from local to global shape. For example, this type of information might be important for arranging the local surface patches in order to construct the overall shape of an object. Consequently, one might expect distortions in perception of global shape to co-occur. Another interesting research direction might be to take a more ecological approach and investigate how haptic length perception is affected when other object properties, such as compliance (cf. Song et al., 2004) or texture (Corsini and Pick, 1969), come into play.

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Chapter 5

Kinematic cues for active haptic shape perception

Abstract

This study quantitatively investigates the actual movements that observers make when exploring a shape. It addresses the question of how the kinematics of these movements affect perceived shape. It is one of the first studies to do so for real stimuli and for relatively unconstrained exploration. Here, shape perception is understood in terms of observing certain invariant relations between shape parameters and the kinematic properties of the exploratory movements that are made. Observers discriminated the curvature of circular strips. The rotation angle of the finger sweeping the strip is found to increase linearly with the curvature of the stimulus. In addition, observers rotated their finger less on a concave curvature by a constant amount. We show that observers took the rotation angle of the finger as their primary measure of perceived curvature. Observers disregarded the systematic difference in movement patterns between concave and convex stimuli; consequently, they overestimated the radius of the concave strips compared to the convex ones.

Introduction

One approach to the study of haptic shape perception of real objects is to look at the geometric stimulus properties that determine performance. Gordon and Morrison (1982) provide an early study addressing this question. Pont, Louw, and coworkers (1998, 1999, 2000, 2002a, 2002b) offer a more thorough investigation into the geometric cues relevant in haptic shape perception. The general idea behind these studies is that detection and discrimination thresholds expressed in terms of the effective geometric cue should be independent of shape and size.

This study takes a different approach. We address the problem of haptic shape perception by looking at how a shape is actually explored by the fingers and hands. Shape cues that are related to these exploratory movements can be identified. By quantitatively analyzing exploratory movements and relating movement characteristics to observers' responses, we can investigate if and to what extent perceived shape is affected by the way that the shape is explored.

One could argue that the strength of approaching the problem of haptic shape perception by investigating relevant geometric cues lies in the fact that no reference is made as to how the fingers actually explore the shape. Discrimination performance is explained entirely in terms of stimulus properties; any findings will thus have a high level of generalizability. However, an explanation purely in terms of physical stimulus characteristics cannot be the whole story, since perceptual abilities are also tied to the characteristics of the sensory apparatus. For example, a number of researchers have noted a critical interaction between the nature of movements of the hands exploring a haptic stimulus and the stimulus property to be perceived (e.g., Davidson, 1976; Gibson, 1962; Klatzky and Lederman, 2003; for an overview, see Appelle, 1991).

Insights into mechanisms at work in haptic shape perception may be gained from studies using haptic interfaces (e.g., Drewing and Ernst, 2006; Henriques and Soechting, 2003). Although valuable in identifying possible shape cues, the fundamental drawback of this approach lies in a reduced set of cues that are predefined by the experimenter. (The machine must be manufactured and programmed). The question then remains as to how exactly observers explore real objects and what shape cues they use. The difficulty of the problem lies in measuring movements for natural exploration of real objects. In the present research, the possible quantitative relation between characteristics of exploratory movements and an observer's shape percept for relatively unconstrained exploration of real objects will be addressed. As such, this is one of the first studies to do so (cf. Sanders and Kappers, submitted; Wong, 1977).

First, we briefly discuss studies of geometric stimulus properties that relate to active dynamic touch (i.e., observers actively moving their fingers across the surface of an object), as this manner of touch is investigated in the present study. Taken together, the two studies by Pont et al. (1998, 1999) show that discrimination of circularly shaped strips is first and foremost based on a comparison of slope differences (Figure 1). Consider two strips (thick lines) of different radii (R_1 and R_2). In the left panel, the two strips have the same scanning length L, whereas in the right panel, they have the same slope difference ϕ. Obviously, radius, scanning length, and slope difference are related parameters: The radius of a circular segment equals the ratio of the scanning length and the slope difference. If observers relied entirely on the slope difference over the stimulus as their geometric shape cue, then they would judge the two strips in the right panel as equally curved. This is not the case, as observers can generally still perceive a difference in radius. However, the important observation is that there is a clear bias towards overestimating the radius of the lower strip (or conversely, underestimating the radius of the upper strip). In other words, the perceptual difference between the two radii is much smaller for the strips in the right panel compared to the strips in the left panel. This is in the direction of the observed slope differences. Thus, the scanning length is taken into account only partially; other geometric cues contribute marginally, making the slope difference over the surface the primary cue (Pont et al., 1999).

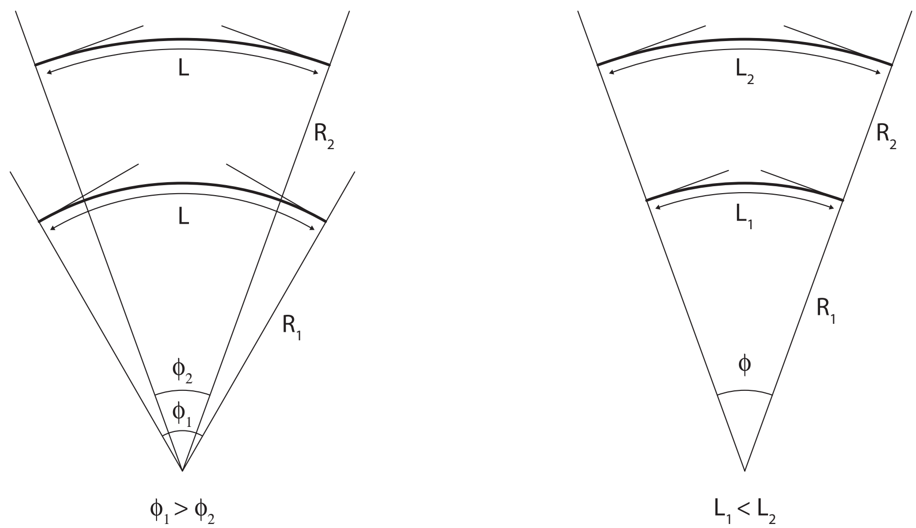

Figure 5.1. Geometric curvature cues. Observers discriminate the radii of the two circular strips (thick lines) by sweeping their finger across the strips' surfaces.

Louw et al. (2000, 2002a, 2002b) provide an extension to more generic shapes. The shapes used in these studies were strips with Gaussian height profiles and strips with second-order derivatives of Gaussian profiles (Mexican hat-like shapes). These stimuli have varying curvature, whereas the circular strips used by Pont et al. (1998, 1999) have constant curvature (curvature is the local change of the slope). Results from all three studies point out that, for both detection and discrimination, the effective cue is the maximum slope, even when different shapes are being discriminated. A minor but unquestionable role is played by another geometric cue, the maximum local curvature.

A starting point for incorporating properties of the hand and hand movements into our understanding of haptic shape perception is offered by Hayward (2008). The author describes a number of possible cues related to the mechanics of contact between finger and circular shapes. The theoretical perspective is that shape perception is understood in terms of observing certain invariant relations between shape parameters and the properties of the contact surface between finger and object.

We now consider what cues may play a role in the present experiment. Kinematic cues involve the displacement of the contact surface between finger and object when the finger moves over the surface. In case of stationary objects and scanning lengths considerably larger than the fingertip's width (as in the present study), slip between finger and object must occur. We can then identify two extreme cases (Figure 2).

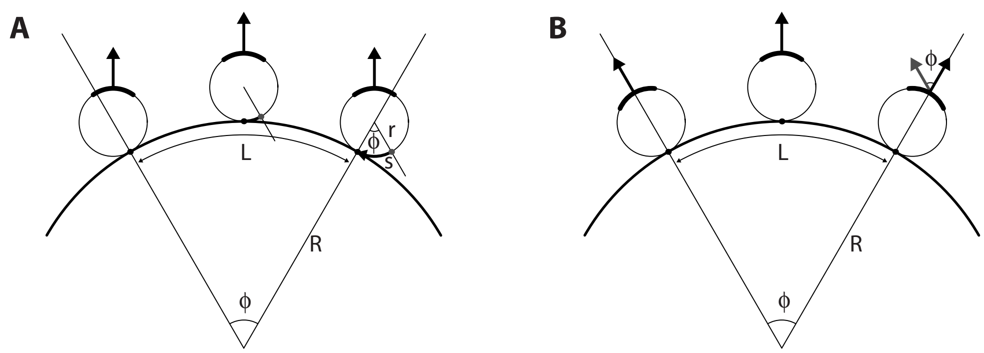

Figure 5.2. Kinematic curvature cues: (A) contact shift, and (B) rotation angle of the finger. Arrows indicating the orientation of the finger are drawn as vectors perpendicular to the nail. Note that radii used in the experiment are much larger than depicted.

First, if the finger keeps the same orientation while moving over the surface, the contact surface shifts across the fingertip (Figure 2A). The contact shift s is related to the radius of the object (convex or concave), which can now be expressed as an invariant CS:

$$\text{CS:} \quad \frac{1}{R} = \frac{s}{rL} \qquad (5.1),$$

where R is the object's radius, L is the scanning length, and r is the effective finger's radius (Hayward, 2008: invariant K2). If the contact shift is larger, then the radius R of the surface must be smaller, given fixed scanning lengths. A study by Dostmohamed and Hayward (2005) supports the idea that the contact shift may indeed be a powerful source of information used by observers.

Second, if the contact point is kept stationary on the fingertip, then the finger will rotate along with the surface normal (Figure 2B), and another invariant, FR, can be identified:

$$\text{FR:} \quad \frac{1}{R} = \frac{\phi}{L} \qquad (5.2),$$

where the rotation angle of the finger is the same as the slope difference ϕ. This invariant may be especially suited for larger objects (cf. Klatzky and Lederman, 2003, p.156: contour following). Invariant FR was not described by Hayward (2008), probably because the author focused on small shapes having low curvatures.

The two exploration modes associated with invariants CS and FR span the range of all possible finger movements when slip is occurring. Interestingly, for invariant CS, the object's radius is observed from cutaneous input, proprioception being needed for controlling the movement. On the other hand, for invariant FR, the radius is observed from proprioception, cutaneous input being needed for movement control. Note that the slope difference over the surface can be sensed from both the contact shift and the rotation angle of the finger, so that this geometric cue is in agreement with either invariant.

Next, we hypothesize biases in perceiving an object's shape to occur, because there are systematic errors in movement patterns of the finger across the object's surface. For example, in the case of invariant CS (Figure 2A), the observer might not fully succeed in keeping the finger in the same orientation or, in the case of invariant FR (Figure 2B), in keeping the contact surface on the same spot on the skin. If we call the rotation angle of the finger θ, Equations (1) and (2) then become:

$$\text{CS:} \qquad \frac{1}{R_p} = \frac{s(\theta)}{rL} \qquad (5.3),$$

and

$$\text{FR:} \qquad \frac{1}{R_p} = \frac{\theta}{L} \qquad (5.4),$$

where R_p represents the perceived radius. The contact shift s now depends on the rotation angle as well: It is shortened by an amount $r\theta$ compared to the shift for θ equal to zero. To summarize, biases in perceiving an object's shape arise, because (1) actual exploration movements do not match exploration movements required for the particular invariant relation observed, and (2) this discrepancy is not (fully) taken into account by the observer.

A recent study by Van der Horst and Kappers (in press) describes an experiment that might enable us to test whether observers rely on a particular kinematic invariant to perceive shape. Participants had to compare a convex with a concave circularly shaped strip and decide which of the two had the larger radius. They found biases indicating that a convex strip systematically appeared flatter than a concave strip of the same radius (Van der Horst and Kappers, in press, Experiment 1: weakly curved stimuli). The crucial observation here is that the geometry of the convex strip is the same as that of the concave strip, apart from a sign. Thus, it is not possible to explain biases in terms of geometric cues: We must look at the way in which the strips are actually being explored by the finger. Unfortunately, Van der Horst and Kappers (in press) did not record movement profiles of the index finger. In addition, the lengths over which participants scanned the strips were left free, so there might have been confounding effects of systematic differences in scanning lengths between convex and concave stimuli (cf. Figure 1B).

In this paper, we present the same curvature-magnitude discrimination experiment on concave and convex strips as described by Van der Horst and

Kappers (in press). However, we fixated the scanning length by using movement stops, such that the scanning length was exactly the same for every stimulus. Figure 3 shows the stimuli used in the present study. We recorded movement profiles of the tracing index finger, expecting to find biases in the case of fixed scanning length.

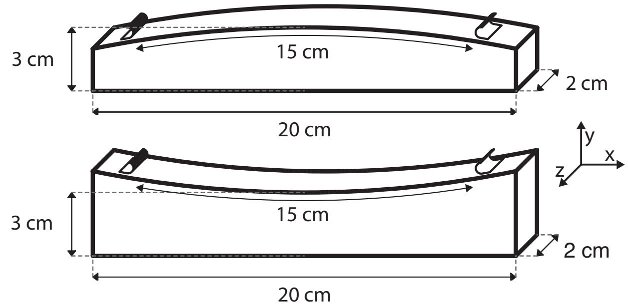

Figure 5.3. Convex and concave reference stimuli along with their dimensions. Convex and concave stimuli are presented one after the other; they are positioned approximately in the participant's frontoparallel plane. Surfaces are touched with the finger pad of the right index finger.

To determine which is the primary cue used by the observer, we proceeded as follows. From the movement data, we extracted the rotation angle of the finger. For every trial, we computed values for invariants according to Equations (3) and (4). Finally, we fitted psychometric curves to the data as a function of physical curvature (geometric cue) and invariants CS and FR (kinematic cues). Thus, we related responses to each of these three possible cues on a trial-to-trial basis. A particular concave and convex strip are judged as equally curved if values of the effective cue are the same for both strips. Therefore, the primary effective cue is that particular cue for which the psychometric curve has a vanishing bias.

Results

Rotation angles as a function of curvature

For every participant, we performed a linear regression on rotation angles and curvature. Figure 4A shows a representative example of the dependence of rotation angles on curvature. Rotation angles are scaled to unit scanning length. The dashed line indicates rotation angles if the finger perfectly follows the curvature of the strip: The rotation angle then equals the slope difference. Two observations can be made. First, the slope of the regression line is between zero rotation (invariant CS) and full rotation (invariant FR). Second, there is a negative y intercept, indicating that the finger systematically rotated less on a concave curvature by a constant amount. In other words, the participant would move on a flat strip as if it were slightly convex. Both observations are schematically illustrated in Figure 4B. These observations resemble the pattern of results averaged over all participants. All regressions were highly significant (ps $< .001$). The mean y intercept was -0.31 and significant ($t_{11} = 3.94$, $p = .002$). The mean slope was 0.37 and significant ($t_{11} = 12.25$,

$p < .001$). Thus, the rotation angle of the finger was only 37% of the slope difference over the surface: The finger followed the curvature of the stimulus only partially.

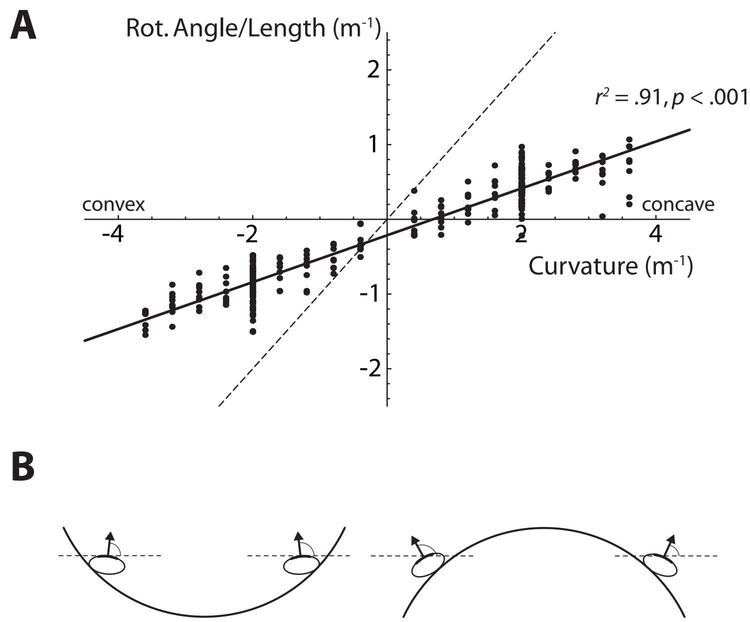

Figure 5.4. Rotation angles as a function of curvature. (A) A typical data set for one of the participants. (B) Schematic illustration of the two observations that the slope of the regression line is less than unity and the *y* intercept is negative.

Psychometric curves as a function of kinematic invariants

As a next step in our analysis, we fitted psychometric curves as a function of curvature differences (ΔC) and differences in observed invariants (ΔCS and ΔFR). First, we computed values for invariants CS and FR from the movement data (see Experimental Procedure). Second, we computed values of ΔCS and ΔFR for every psychometric trial and fitted these to response fractions. Thus, we excluded between-trial variability from analysis, as this variability is unrelated to responses.

Figure 5 shows a representative example of a psychometric curve from one observer fitted as a function of each of the three independent variables. The so-called Point of Subjective Equality (indicated as μ) corresponds to a response score of 50% and represents the combination of concave and convex stimuli that were judged equal when expressed in terms of the respective variable. Figure 6 shows Points of Subjective Equality (PSE) averaged over all twelve observers. The average PSE in terms of curvature difference (ΔC) was 0.48 m^{-1}. This was significantly different from zero ($t_{11} = 3.24$, $p = .008$). The positive bias indicates that a concave strip must be curved more than a convex strip in order to be perceived as equal: A concave curvature appeared flatter than the same convex curvature. The average PSE in terms of contact shift (ΔCS) was 1.15 m^{-1}, which also differed significantly from zero ($t_{11} = 4.83$, $p < .001$). However, the average PSE in terms of finger rotation (ΔFR) was -0.21 m^{-1}. This was not significantly different from zero ($t_{11} = 1.04$, $p = .319$).

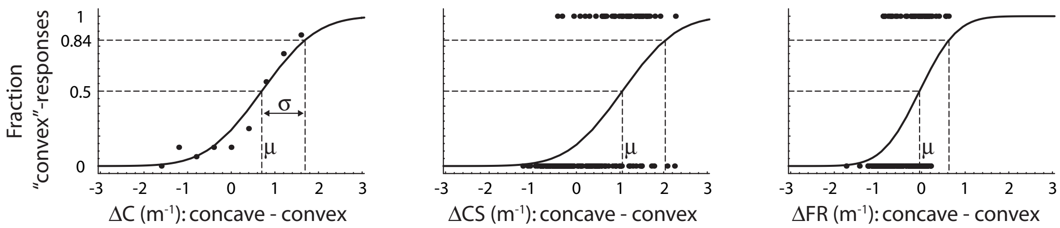

Figure 5.5. Psychometric curves. Representative examples of psychometric curves fitted as a function of curvature differences (*ΔC*) and differences in observed invariants (*ΔCS* and *ΔFR*). All curves are from the same observer, and each consists of 144 2AFC trials. The Points of Subjective Equality are indicated by μ; the average threshold σ expressed in physical curvature in this study was 0.81 m^{-1} (SD = 0.25 m^{-1}).

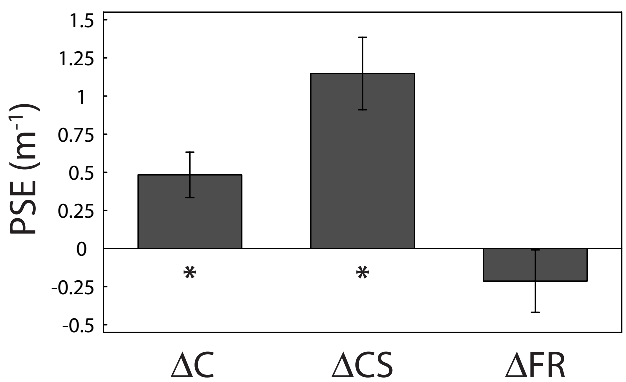

Figure 5.6. Points of Subjective Equality. Average PSEs expressed in terms of curvature differences (*ΔC*) and differences in observed invariants (*ΔCS* and *ΔFR*). Error bars represent standard errors of the mean. PSE-ΔC and PSE-ΔCS differed significantly from zero, *p*s < .01.

Movement speed

We tested whether there were any systematic differences in movement speed between concave and convex stimuli. To exclude effects of curvature, we computed average movement speeds only for those trials in which both the concave and convex strips had reference curvature (N = 16 trials per observer). There was considerable variation between observers. Movement speeds could differ by more than a factor of three (*Mdn* = 22.3 cm/s, *Min* = 12.5 cm/s, *Max* = 46.4 cm/s). Paired-samples *t* tests indicated that concave speeds were slightly higher than convex speeds (t_{15} = 5.74, p < .001) for only one observer. For all other observers, speed differences were non-significant (*p*s > .141).

Discussion

Surprisingly, we found biases indicating that a concave curvature appeared flatter than a convex curvature of the same radius. Scanning lengths were fixated through the use of movement stops, so that the two types of strips were exactly the same in terms of geometric cues, apart from a sign (Pont et al., 1998, 1999). Therefore, finding these biases calls for an analysis of finger movements across the stimuli. From recordings of the location and orientation of the index finger tracing the stimulus, we could deduce that there were systematic differences between concave and convex curvatures. Regressions of rotation angles on curvature showed that observers on average rotated less on a concave curvature by a constant amount (negative *y* intercepts).

In the Introduction, we argued that a possible mechanism for haptic shape perception must be based on observing kinematic invariants. Kinematic invariants are fixed relations between the curvature of the surface and certain kinematic characteristics of exploratory movements, independent of the speed of exploration. In the case of slip occurring between finger and object, two invariants (CS and FR) can be identified that are related to the shift of contact across the fingertip or the rotation of the finger. Thus, biases would be a direct consequence of systematic differences in exploratory movements between concave and convex stimuli that are not (fully) accounted for by the observer (see Equations (3) and (4)).

The results indicate that biases expressed in terms of invariant FR disappear, whereas biases in terms of invariant CS remain significant (Figure 6). By fitting psychometric curves as a function of differences in observed kinematic invariants, we related responses to observed invariants on a trial-to-trial basis. The vanishing bias for invariant FR shows that this invariant is the effective cue for curvature discrimination in our experiment: A zero bias indicates that concave and convex strips were judged as equally curved if the magnitudes of invariant FR were the same. We conclude that observers take rotation angle of the index finger moving across the stimulus as their primary measure of perceived curvature.

One may ask whether the brain can use invariant CS as an additional source of information. Perhaps the two cues can be integrated to form a single curvature percept according to their weighted average (cue weighting). The rotation of the finger followed the curvature of the stimulus for only 37% on average, so there must have been also a nonzero shift of contact when moving over the stimulus. Thus, although invariant FR is the primary cue, the question remains how cutaneous input from the shift of contact surface between finger and stimulus is treated.

We think it is reasonable to assume that the contact shift is simply ignored, since exploratory movements related to invariants CS and FR form two opposite extremes. Consider a strip with a certain radius. As one proceeds from full rotation with the curvature of the stimulus through partial rotation to maintaining the finger oriented horizontally, the rotation angle of the finger decreases whereas the contact shift increases. Correspondingly, perceived curvature (see Equations (3) and (4)) decreases when observing invariant FR but increases when observing invariant CS. The two cues are not independent. Thus, depending on the particular exploratory procedure adopted, one cue or the other will be selected; the other must then be ignored.

Other sources of information that are not considered include local skin deformation patterns and their related static invariants (cf. Hayward, 2008). The experimental difficulty lies in measuring local skin deformation for the human finger under normal conditions (e.g., Pawluk and Howe, 1999). At present, one would have to divert to haptic devices (e.g., Provancher et al., 2005; Wang and Hayward, 2006) and simulations of fingertip deformations (e.g., Serina et al., 1998) to understand the exact role of skin deformations in shape perception. The studies by Louw et al. (2000, 2002a, 2002b) for real stimuli suggest that skin deformation patterns might indeed play a significant role in haptic shape perception of real stimuli. As such, they may also cause biases. Clearly, local skin deformation patterns are different for concave and convex surfaces of the same radius due to the convex shape of the fingertip.

In this study, we focused on circular strips. However, the emphasis on surfaces with constant curvature does not mean a loss of generalizability. In mathematical terms, smooth surfaces can be locally approximated by doubly curved surfaces that are either egg-like or saddle-like (e.g., Hilbert and Cohn-Vossen, 1932; Porteous, 2001). Any shape can thus be rendered as an ordered array of these elementary surface patches. The case of kinematic invariants for circular strips may be easily extended to shape perception in general. The observer would be sensing instantaneous values of the invariants while moving the finger across the object's surface. Invariants CS and FR are expressed as the ratios of contact velocity and scanning velocity and of angular velocity and scanning velocity, respectively.

The present study provides a quantitative investigation into how observers explore real shapes by touch and how the manner of exploration affects perceived shape. In a curvature-magnitude discrimination experiment, observers compared concave and convex strips and then decided which felt more flat. We constrained exploration as less as possible. When sweeping over the circular strips, the finger partially followed the curvature of the surfaces. The rotation angle increased linearly with the curvature of the stimulus. The results indicate that observers took the rotation angle of the finger as their primary measure of perceived curvature.

Experimental Procedure

Participants

The participants consisted of eleven males and one female in their 20s and early 30s. They were all right-handed. Participants were naive as to the aims and design of the experiment. Informed consent was obtained from each participant. They were paid for their efforts.

Stimuli and setup

Stimuli were cylindrically curved strips made of PVC (see Figure 3). The same strips were used by Van der Horst and Kappers (in press). For circular strips, local curvature is constant and equal to the reciprocal of the strip's radius. Curvatures for both the concave and convex strips ranged from 0.4 m^{-1} to 3.6 m^{-1} in steps of 0.4 m^{-1}. The base-to-peak height was 3 cm for all strips. The heights of the end points varied slightly with the curvature of the stimulus. We placed circular aluminum stops (height 7 mm) on the curved surfaces such that the scanning length was always 15 cm, irrespective of the stimulus' curvature. The scanning path was positioned symmetrically on the surface. The movement stops did not convey any information about the curvature of the stimulus.

Upon stimulus presentation, curved strips were put in a thin cardboard placeholder fixed on a table (height 75 cm), behind which the participant was comfortably seated. Stimuli were oriented to be in the participant's frontoparallel plane, with their midpoints approximately in the sagittal plane passing through the right shoulder. Participants touched the stimuli with the finger pad of the index finger of their right hand (volar surface of distal phalanx). Thus, when tracing the stimuli, participants moved their index finger from left to right and vice versa. The distance between the participant and the table edge was adjusted according to their liking. Generally, when touching the stimuli, the angle between upper arm and forearm was more than 90 degrees. Participants could move their arms freely when tracing the stimuli, but they did not rest their elbow on any support or touch the table's edge. Talcum powder was used to reduce friction between skin and stimulus surface.

Design and procedure

Measuring psychometric functions

We measured psychometric functions according to the two-alternative forced-choice (2AFC) method using constant stimuli to determine which concave and convex curvatures were perceived as equal. Each trial proceeded as follows. Participants were presented with a concave or a convex curvature. They placed their right index finger on the curved surface against the left movement stop. After a signal from the experimenter, they traced the stimulus four times, going back and forth twice. When they lifted their hand, the experimenter quickly placed the second stimulus, having an opposite curvature, in front of them. The participants positioned their finger against the second curvature's left stop and, after a signal from the experimenter, traced the next stimulus four times. They then had to decide which of the two stimuli felt more flat. Participants were instructed to make smooth movements, to touch the stops when moving across the stimuli, and to keep their finger perpendicular to the stimulus (parallel to the z axis, Figure 3) as much as possible.

The curvature difference ΔC between the concave and convex curvature (Cv – Cx) served as the independent variable.[*] Curvature differences sampled were ± 1.6, ± 1.2, ± 0.8, ± 0.4 and 0 m^{-1} at a reference curvature of 2 m^{-1}. A curvature difference of, for example, +1.6 m^{-1} corresponded to four possible combinations: Cv-3.6, Cx-2; Cx-2, Cv-3.6; Cv-2, Cx-0.4; or Cx-0.4, Cv-2 m^{-1}. Thus, the two curvatures were presented in both time orders to avoid response biases, and the reference curvature could be concave or convex to prevent any learning effects. The set containing the four possible stimulus combinations for each of the nine curvature differences was presented four times, amounting to a total of 144 trials per psychometric curve. Trials were independently randomized within each of the four blocks. A psychometric curve was measured in two sessions on separate days, with each session containing half of the trials. Every session started with a few practice trials. Participants were blindfolded throughout the experiment. A single session lasted about 0.75 hours.

[*] Note that in the study by Van der Horst and Kappers (in press), curvature differences were defined in the opposite way: convex – concave.

Measuring location and orientation of the index finger

Movement profiles of the index finger were recorded with an Optotrak Certus system (Northern Digital Inc., Canada). Three position markers were placed on the nail of the index finger, enabling us to measure not only the finger's location but also its orientation. In addition, position markers were fixed to the setup such that we could retrieve coordinates of the stimulus' curved surface. Three-dimensional spatial coordinates of the position markers were read at a frequency of 100 Hz. The vector perpendicular to the plane defined by the three markers on the nail represented the finger's orientation. The orientation of this plane corresponds to the orientation of the nail only in rough approximation. This is not a problem, since in the end we are only interested in the change of orientation. Finally, the location of the index finger was defined as the average location of the three position markers.

Data analysis

Rotation angle of the finger

We extracted two main parameters for analysis. First, we projected the finger's location on the space curve passing through the stimulus' curved edge (i.e., the point on the stimulus with the shortest distance to the finger's location). A location parameter specifying the position of the index finger was then defined as the distance from the projection to the midpoint of the stimulus (measured along the curved edge). Second, the vector perpendicular to the nail was projected on the *xy* plane, and the angle between the projected vector and the horizontal was used to parameterize the finger's orientation.

Figure 7A shows typical orientation-location profiles for a concave and a convex reference curvature. The orientation of the finger with respect to the horizontal is given as a function of the location on the stimulus. The participant starts at the left end point and goes back and forth over the stimulus twice. At the left and right end points, there is a sudden change in the finger's orientation, indicating that there is a more or less instantaneous tilt of the finger at movement onset due to friction between the skin and the surface (Figure 7B). Note that the change in orientation when moving across the stimulus is less for the concave curvature than for the convex one. Furthermore, the full range of the location parameter is less than the distance between the movement stops (15 cm). This is because the effective scanning length is shortened by the width of the fingertip.

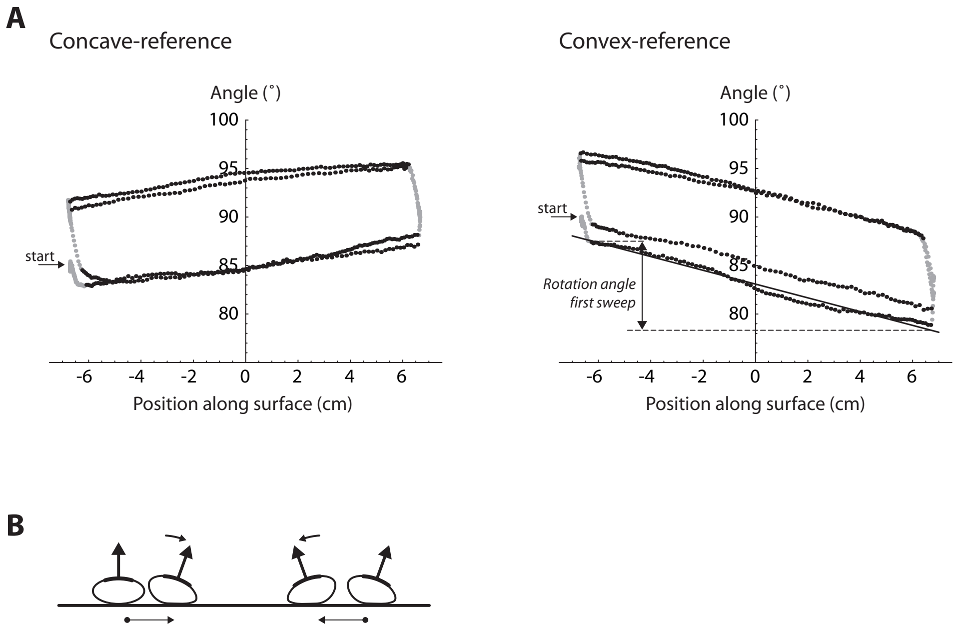

Figure 5.7. Extracting rotation angle of the finger. (A) Typical orientation-location profiles for concave and convex reference curvature. (B) Schematic illustration of the sudden change in nail orientation at movement onset or when movement direction changes.

Next, to exclude any effects due to friction or due to the movement stops, we extracted those parts from the orientation-location profiles that the finger was moving across the stimulus. We proceeded as follows. First, we computed velocity profiles by taking the three-point derivative of location-time profiles (Press et al., 2002, Chapter 5). Sweeps were defined as those parts of the movement profiles for which the speed reached a certain threshold. Values for speed thresholds were a fixed fraction of the peak velocity for the respective trials. For all observers except one, a fraction of one quarter of the peak velocity yielded reliable sweep extractions; for one observer, this was a fraction of one third. We ended up with four sweeps for each trial (see black parts in Figure 7A).

For each sweep, we calculated the rotation angle by performing a linear regression and then calculating the orientation difference according to the regression equation. Thus, for each sweep, we have a location range (scanning length) and a change of orientation over this range. The mean scanning length and rotation angle for the four sweeps represents the scanning length and rotation angle for each particular trial. We also performed a full analysis by defining rotation angles as the orientation difference between the first and the last recordings for each sweep. This method yielded the same pattern as the overall results.

Points of Subjective Equality

We used cumulative Gaussian distributions as psychometric functions:

$$PF(x) = \tfrac{1}{2} + \tfrac{1}{2} erf\left(\tfrac{x-\mu}{\sqrt{2}\sigma}\right) \qquad (5.5),$$

where *erf* is the Error Function. The shape of this psychometric function is determined by two parameters: μ, which represents the location of the curve, and σ, which indicates its shallowness (Figure 5). They are interpreted as follows: μ is the point at which the response score is 50% and is assumed to represent the observer's Point of Subjective Equality (PSE); σ is the difference between the 50% point and the 84% point and is taken as a measure of the discrimination threshold. Psychometric curves were fitted to the data by assuming that responses were generated by Bernoulli processes (for computational details, refer to Wichmann and Hill, 2001).

Psychometric curves were fitted as a function of curvature differences (ΔC) and as a function of differences in observed invariants (ΔCS and ΔFR). Independent variables ΔCS and ΔFR were defined in the same way as the curvature difference ΔC: concave – convex. We computed invariants CS and FR according to Equations (3) and (4); the contact shift is reconstructed from the rotation angle:

$$s(\theta) = r\left(\frac{L}{R} - \theta\right) \qquad (5.6).$$

The assumption underlying Equation (6) is that the finger can be regarded as effectively circular with some radius *r* over the range of contact shifts observed. This seems reasonable, since contact shifts were small compared to the perimeter of the index finger. Both the mean rotation angle and the mean scanning length (θ and L, respectively) were taken into account on a trial-to-trial basis.

Instead of nine discrete values resulting from sixteen repetitions, the psychometric function is now sampled at 144 points in the case of independent variables ΔCS and ΔFR. The independent variable forms a continuum, because we extracted it from movement data. Response fractions for each of these 144 points are either zero or one.*

It is essential that values for ΔCS and ΔFR were computed on a trial-to-trial basis. In every psychometric trial, observers compared two stimuli. One therefore expects that movement profiles for these two curvatures were tailored to each other, such that the participant was able to make a good comparison of their curvatures by observing one of the invariant relations. On the other hand, there was obviously no principal need for a tight correlation between movement characteristics of different psychometric trials. For example, an observer might move over the first stimulus in such a way that the finger follows the curvature of the surface quite well. When exploring the second stimulus, the observer would again rotate with the surface's curvature. In another trial, the observer might follow the curvature of the stimulus to a lesser extent; consequently, the same exploratory movement would be adopted for the second stimulus. Thus, by analyzing on a trial-to-trial basis, we prevent any

* It is perfectly valid to fit the psychometric curve on binary data in Figure 5. Take, for example, the psychometric curve as a function of the curvature difference in Figure 5 and consider the response fraction at 1.6 m^{-1}. This point is the result of 14 "convex"-responses and two "concave"-responses and its coordinates are therefore (1.6; 0.875). However, we could as well replace this single point with 14 points having coordinates (1.6; 1) and two points having coordinates (1.6; 0). If we replace all other points with their binary counterparts and fit the curve again, this time as a function of 144 data points, we obtain the same fit parameters.

possible correlation between movement data and subjective estimates to be obscured by between-trial variability. To put this in more concrete terms, we account for the variability in rotation angles for fixed curvatures as shown in Figure 4A.

Error analysis location parameter

Recall that the location parameter is defined as the perpendicular projection of the average location of the three nail markers on the stimulus' surface. This is the only feasible way to parameterize the location of the finger. However, the projection might not correspond precisely to the actual location of the finger (i.e., the contact point between finger and surface). Consequently, small errors in the measured scanning length might be introduced. If the finger fully rotates with the surface's curvature (Figure 2B), then the scanning length is measured correctly. However, if the finger does not fully rotate or maintains a constant orientation, then a systematic error in scanning length between the two types of stimuli occurs. Considering Figure 2A, if we draw a line through the center of the circular strip and the midpoint of the nail, the intersection of this line with the strip's surface is the location as we measure it. For convex stimuli, the measured scanning length is then slightly smaller than the actual scanning length; for concave stimuli, the measured scanning length is slightly larger. These systematic errors are in the same direction as the reported effects: The results show that the rotation angle per unit scanning length was smaller for concave stimuli. The difference between the measured and actual scanning lengths depends on the scanning length itself as well as on the radius of the surface and the effective radius of the finger. The latter we do not know.

To estimate the magnitude of this effect, we measured the difference in scanning length between zero rotation and full rotation for three observers. We asked them to place their index finger against the left and right end stops on both the concave and convex reference curvatures. They oriented their finger either horizontally or parallel to the surface. Overall, the average difference in scanning length between zero and full rotation was 4.5 mm (SD = 1.5 mm). This corresponds to 3.5% of the overall average scanning length (130 mm). Next, we fitted the psychometric curves again, adding 4.5 mm to all convex scanning lengths and subtracting this amount from all concave lengths. The pattern of results remained the same: The average PSEs in terms of contact shift (ΔCS) and finger rotation (ΔFR) were 1.09 m^{-1} and -0.17 m^{-1}, respectively. Note that this is a conservative error estimate. Since participants rotated with the curvature of the surface for 37%, the systematic errors in convex and concave scanning lengths were maximally 63% of 4.5 mm.

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Appendix

In this Appendix, we will discuss in more detail the relation between the present results, the results of Chapter 4 and the study by Van der Horst and Kappers (in press).

The first question that has to be answered is how to reconcile the findings of this chapter with those of Van der Horst and Kappers (in press). We were surprised to find opposite curvature biases in Chapter 5: Van der Horst and Kappers (in press) found convex curvatures to be judged more flat. Two explanations are possible. In the study by Van der Horst and Kappers (in press), scanning length was left free. As we have explained, the length over which stimuli are scanned influences perceived curvature. Therefore, biases reported in Van der Horst and Kappers (in press) might have been compound effects of biases due to differences in scanning length (convex scanning lengths being shorter) and biases due to differences in rotation angle of the tracing index finger. The former bias must then have been larger in magnitude than the latter one. Although we used the same stimulus set, there were also slight differences in setup. In our study, stimuli were positioned in front of the right shoulder and observers could not rest their arm on any support while tracing the stimulus. In the study by Van der Horst and Kappers (in press), stimuli were positioned on the body midline and observers rested their forearm on a table. Thus, observers might have relied more on movement of the wrist to trace the curved strips compared to Chapter 5. Therefore, a second possible explanation is that the difference in setup posed different constraints on movements of the right arm, which resulted in opposite biases. Observers in the study by Van der Horst and Kappers (in press) would then have rotated their finger less on a convex stimulus compared to a concave stimulus of the same radius.

The second question pertains to the finding of non-significant speed differences between concave and convex stimuli in this chapter, whereas we found significant speed differences between convex and concave surfaces in Chapter 4. In the latter study, we found that observers estimating lengths of curved pathways traced convex pathways slightly slower than concave pathways. For only one observer (out of 12 observers) in this chapter did we find a significant speed difference; however, this speed differences was in agreement with the results of Chapter 4. In a length discrimination experiment, time is a critical variable and consequently so is movement speed. On the other hand, in our curvature-magnitude discrimination experiment, rotation angle of the finger proved to be a critical variable. Consequently, small variations in movement speed will not affect perceived curvature. There is no principal need to match –perceived-- concave and convex speeds when judging curvature. Of course, scanning lengths have to be taken into account as well, but in the experiments presented in this chapter it was fixated through the use of movement stops. Observers were not informed about scanning lengths being the same for all stimuli, but they may implicitly have assumed that this was the case. Thus, it would be interesting to test whether observers more closely control movement speed in a way similar to Chapter 4 if scanning length was left free.

A third question is whether the present results agree with the coupling between curvature perception and length perception as proposed in Chapter 4. We suggested an effect of perceived curvature on perceived movement speeds across convex and concave surfaces as a possible explanation for the results reported in that chapter. This explanation is based on curvature biases having the direction reported in Van der Horst and Kappers (in press). However, one may argue that the conditions of the present experiment match the conditions of Chapter 4 more closely. In this chapter, we controlled for scanning length and clearly this was also the case in Chapter 4. Thus, curvature biases reported in Chapter 5 may provide a better comparison. These curvature biases are opposite in sign to the biases reported in Van der Horst and Kappers (in press) and therefore they do not agree with the hypothesis proposed in Chapter 4.

However, we feel that it is still too early to discard the hypothesis put forward in Chapter 4. Again, there were small differences in stimuli and setup. For example, the reference radius in the present experiment was 50 cm, whereas the surfaces used in the preceding chapter had radii of 10 and 20 cm. Moreover, cutaneous input in the experiments of Chapter 4 was essential for signaling the end points of the strip. These tiny differences in stimuli and setup might have provoked different movements or even the use of different curvature invariants in Chapter 4. (The rejection of the contact point hypothesis argues against the use of invariant CS but does not disprove it.) Thus, taking these differences in setup into account, perception of curvature and distance may still be consistent with each other. Further investigations into the relation between curvature perception and distance perception are needed.

Finally, we remark that the answers to all three questions raised above suggest that the way observers move over the stimuli may be easily influenced by the stimulus set, the particular setup used, or the setup's location. Of course, this supposition also has to be further investigated but one may wonder to what extent results obtained in Chapters 4 and 5 would then be generalizable beyond the particular setup used in these two chapters. In everyday exploration of objects, observers may transcend these biomechanical constraints related to the setup by observing various cues successively and applying different manners of touching the stimuli accordingly, or by orienting and positioning the object relative to the body in different ways. At this point, however, it is important to keep in mind that what these studies in any case show is that for real stimuli and active touch, an observer's percept can follow directly from properties of arm movements: As such, these studies add to our understanding of haptic perception.

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Chapter 6

Factors affecting the haptic filled-space illusion for dynamic touch

Abstract

In the haptic filled-space illusion for active dynamic touch, observers move their fingertip across an unfilled extent or an extent filled with intermediate stimulations. Previous researchers have reported lengths of filled extents to be overestimated, but the parameters affecting the strength of the illusion are still largely unknown. In the current research, we show that the illusion persists when intermediate stimulations do not provide information about the extent's length. In addition, the results show that the strength of the illusion increases with the number of filler elements. In contrast with earlier research, we control for movement speed differences between filled and unfilled extents. The results suggest that the strength of the illusion is independent of the overall average movement speed. Insight into factors affecting the strength of the illusion may provide a better understanding of the kinematic mechanisms underlying haptic length perception.

Introduction

In the long-known visual filled-space illusion, observers judge a linear extent that is filled with discrete filler elements to be longer than an unfilled extent of the same length (also known as Oppel-Kundt illusion; e.g., Coren and Girgus, 1978). Although less studied, various haptic analogues of the filled-space illusion have been identified over the years (Craig, 1931; Dresslar, 1893; Jaensch, 1906; Parrish, 1893; Révész, 1934; Suzuki and Arashida, 1992). Filled extents in the haptic case include punctured cardboard cards, raised-line gratings, wooden laths with holes cut out on the edges, and so on.

One might find the coexistence of illusory effects of filled extents in both the visual and haptic modality to be striking: This suggests, for example, that the haptic illusion might be mediated through the visualization of what is being touched and would thus be essentially visual in nature. However, geometric illusions have been reported for congenitally blind people who have had no visual experience (e.g., Heller et al., 2002). Moreover, a number of researchers have argued that close ties exist between the way a haptic stimulus is explored by the hand and the way it is perceived (e.g., Davidson, 1976; Gibson, 1962; for an overview, see Appelle, 1991). In the case of the haptic filled-space illusion, filled extents are overestimated when participants actively move their finger across the stimuli (active dynamic touch), whereas filled extents are underestimated when stimuli are impressed onto the skin of the forearm (static touch; Craig, 1931; Parrish, 1893). This raises the question regarding the nature of intermediate stimulations in the haptic filled-space illusion.

In this study, we investigate the haptic filled-space illusion for active dynamic touch. Previously, Révész (1934) mentioned the existence of the illusion but did not provide quantitative data. Suzuki and Arashida (1992) measured Points of Subjective Equality in a discrimination experiment, but used a reference length of only a single wavelength. Likewise, Jaensch (1906) tested filled extents of a single wavelength in a reproduction experiment. Only the study by Dresslar (1893) provides indications concerning parameterization of the illusion. He used punctured cardboard cards, with the results suggesting that overestimations increase with the number of punctures. In all of these studies, the movement speed of the tracing finger was not controlled for.

Increasing evidence shows that haptic length perception is based on observing kinematic variables. For example, observers who were instructed to trace linear extents at high speeds judged these lengths to be shorter than the same extents traced at considerably lower speeds (Hollins and Goble, 1988; Von Skramlik, 1933; Wapner et al., 1967). In the case of free exploration, linear extents oriented radially from the body were judged to be longer than extents oriented tangentially; at the same time, it was observed that the extents that were judged longer were traced at slightly lower speeds, which led to a corresponding increase in movement time (Armstrong and Marks, 1999; Wong, 1977). Similarly, convex lengths were overestimated compared to concave lengths; again, these overestimations correlated with differences in movement time that were, in turn, due to slight speed differences between the various surface types (Sanders and Kappers, submitted). Thus, one explanation for the haptic filled-space illusion is that filled extents are traced at slightly lower speeds.

However, findings of Dresslar (1893) suggest otherwise. He succeeded in conducting his experiment under conditions of passive dynamic touch (cardboard being moved underneath stationary fingertip), thus being able to make movement times exactly the same. Interestingly, he reported the illusion to persist. However, one should be cautious in comparing active and passive touch conditions.

A confounding factor in all of these studies is that the end points of extents are the same as the filler elements. Thus, for unfilled extents, the puncture the finger comes across after having passed over the first one is the puncture marking the end point. On the other hand, for filled extents, the observer has to move the index finger one wavelength beyond the last puncture to be sure that the extent has ended. Consequently, this might lead the participant to overestimate filled extents. If so, doubts are cast on whether the illusion is actually one of overestimation of filled *space*. Interestingly, none of the studies has noted this possible end point effect, while biases reported are of the same order of magnitude (cf. Suzuki and Arashida, 1992).

In this research, we investigated the filled-space illusion in a more thorough and systematic way than has previously been done. In Experiment 1, we investigated whether the overestimations of filled extents compared with unfilled extents increase with the number of filler elements. Filled extents are defined to be extents with discrete intermediate stimulations at the fingertip tracing the stimuli. We also investigated whether overall movement speed has an effect on the strength of the illusion. In Experiment 2, participants directly compared filled extents of different filler densities. None of the studies mentioned have tested direct comparisons of filled extents. If we find that overestimations in Experiment 1 increase with the number of filler elements, we expect filled extents to be overestimated compared to less densely filled extents in the case of direct comparisons in Experiment 2. In both experiments, we excluded end point effects by using easily identifiable end points that are clearly distinguishable from filler elements. In addition, movement speeds within a trial have been made the same through the use of a metronome.

Materials and methods

Participants

The participants consisted of five males and two females who were in their 20s or early 30s, all of whom were right-handed. Participants were naive as to the aims and design of the experiments, and were paid for their efforts.

Stimuli and setup

Stimuli were linear extents defined by two small rubber pads glued onto sheets of swell paper (Figure 1A). The rubber pads were spherical caps with a diameter of 8 mm and a height of approximately 2 mm. Between the two rubber pads, we printed raised spatial patterns. These patterns were either gratings with wavelengths (λ) of 4 or 8 mm, or raised blocks. Gratings consisted of raised lines with a width of 1 mm. When moving the finger across the stimuli, end points were readily identifiable. Nine comparison lengths (*L*) were available: 8.8, 9.6, 10.4, 11.2, 12, 12.8, 13.6, 14.4, and 15.2 cm. Note that comparison lengths are multiples of the two wavelengths used, so that the distribution of raised lines is uniform over the entire stimulus.

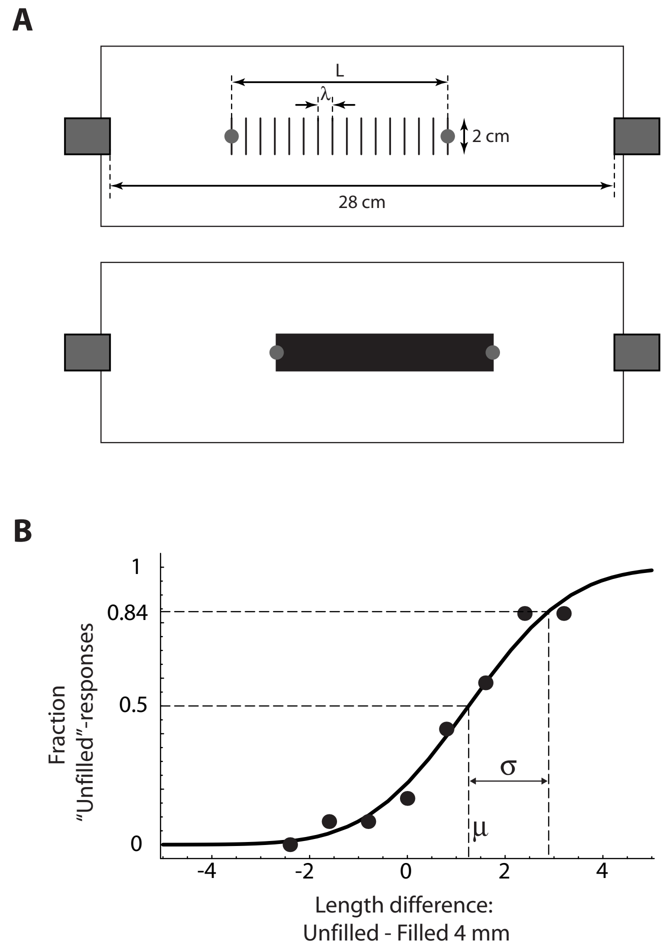

Figure 6.1. (A) Stimuli. Participants moved their index finger between two stops, pacing movement speed through the use of a metronome. End points of filled and unfilled extents were rubber pads that were easily distinguishable from raised filler patterns. Stimulus lengths were randomly displaced to the left or right of the center of the setup by a constant amount. (B) Psychometric curve. A typical example of a psychometric curve obtained in this study is shown. Parameter μ represents the Point of Subjective Equality.

Upon stimulus presentation, swell paper sheets were placed into a stimulus holder that was fixed onto a tabletop. Participants were comfortably seated behind the table, their right shoulder being approximately on the setup's midline. On either side of the stimulus holder, two movement stops were attached (small wooden blocks). The distance between the two movement stops was 28 cm. Stimuli were touched with the fingertip of the right index finger. Observers were instructed to make smooth movements between the movement stops, touching the stimulus on the way. A metronome was ticking at a frequency of 1 or 2 Hz. To induce observers to move with equal speeds across the different stimuli, they were required to synchronize each tick of the metronome with arrival of the index finger at the left or right movement stop.

The midpoint of the stimulus was randomly located 1.25 cm to the left or right of the setup's midpoint to prevent observers from using the locations of the end points as length cues. Talcum powder was used to reduce friction between the finger and the rubber pads. The participants wore soundproof headphones that blocked auditory cues from the finger moving across the grating, while at the same time permitting the ticks of the metronome to pass. Participants were also blindfolded. Halfway through the experiment we replaced the stimulus set to prevent wear-out of the raised-line drawings.

Design and procedure

A trial proceeded as follows. Participants put their right index finger against the left movement stop on the first stimulus sheet and traced the stimulus a number of times, keeping pace with the ticks of the metronome. Participants lifted their hands while the experimenter quickly changed the swell paper sheets, and the participants explored the second stimulus. They then decided which of the two stimuli felt longer. Generally, participants traced a stimulus back and forth three or four times.

We measured psychometric functions according to the two-alternative forced-choice (2AFC) method using constant stimuli. The reference length was 12 cm throughout. The length difference between the two types of stimuli served as the independent variable. In Experiment 1, participants compared raised blocks (U: Unfilled) with gratings (F: Filled), where the length difference is defined as U – F. In Experiment 2, participants compared gratings of wavelengths 4 and 8 mm; the length difference is defined as F8 – F4. For example, a length difference of +2.4 cm corresponded to the following four combinations: U-14.4 cm, F-12 cm; F-12 cm, U-14.4 cm; U-12 cm, F-9.6; and F-9.6, U-12 cm. The two lengths of a stimulus pair were presented in both time orders to avoid response biases, and the reference length was a raised block or a grating to prevent any learning effects. Based on pilot experiments, we anticipated positive biases in all conditions. Therefore, we added a length difference of +3.2 cm to the distribution of sample points to make it appear more symmetric to the observer (Figure 1B).

The set containing the four possible combinations for each of the eight possible length differences was presented three times, amounting to a total of 96 trials per psychometric curve. Trials were independently randomized within each of the three blocks. A psychometric curve was measured in a single session lasting about 1.5 h; sessions were done on separate days. Before commencing the actual experiment, participants were given a practice session to get accustomed to the setup and use of the metronome. Every session started with a few practice trials. Participants were never given any feedback.

In Experiment 1, participants compared raised blocks with gratings of different wavelengths. This experiment consisted of four conditions that were formed by the combination of the two factors of wavelength (4 mm/ 8 mm) and metronome frequency (1 Hz/ 2 Hz). In Experiment 2, participants discriminated lengths of gratings with wavelengths of 8 mm from wavelengths of 4 mm. Two conditions were tested, which corresponded to the factor of metronome frequency (1 Hz/ 2 Hz). To prevent learning effects, the conditions for both experiments were mixed and randomized across observers.

Data analysis

Cumulative Gaussians were used as psychometric functions. The two parameters μ and σ (Figure 1B) are interpreted as the Point of Subjective Equality (PSE) and the discrimination threshold, respectively. All *p*-values reported are two-tailed, unless explicitly stated otherwise.

Results

Experiment 1

Figure 2A shows average PSEs for the four conditions in Experiment 1. Biases were significantly different from zero in all cases, with $t_6 = 3.18$, $p = .009$; $t_6 = 3.61$, $p = .006$; $t_6 = 3.14$, $p = .010$; and $t_6 = 2.79$, $p = .016$ for conditions U-F4/1Hz, U-F4/2Hz, U-F8/1Hz, and U-F8/2Hz, respectively (one-tailed *p*-values). Positive biases indicate that the unfilled extent had to be longer than the grating in order for the two to be perceived as equally long. Thus, observers overestimated lengths of gratings. A two-by-two ANOVA (wavelength x movement speed) with a repeated-measures design showed a significant main effect of wavelength ($F_{1,6} = 6.95$, $p = .039$) but no main effect of movement speed ($F_{1,6} = 0.79$, $p = .409$). There was no significant interaction effect ($F_{1,6} = 0.11$, $p = .752$). Average biases for conditions U-F4 and U-F8 were 1.08 cm and 0.85 cm, respectively. This corresponds to an overall average effect size of 8.4%.

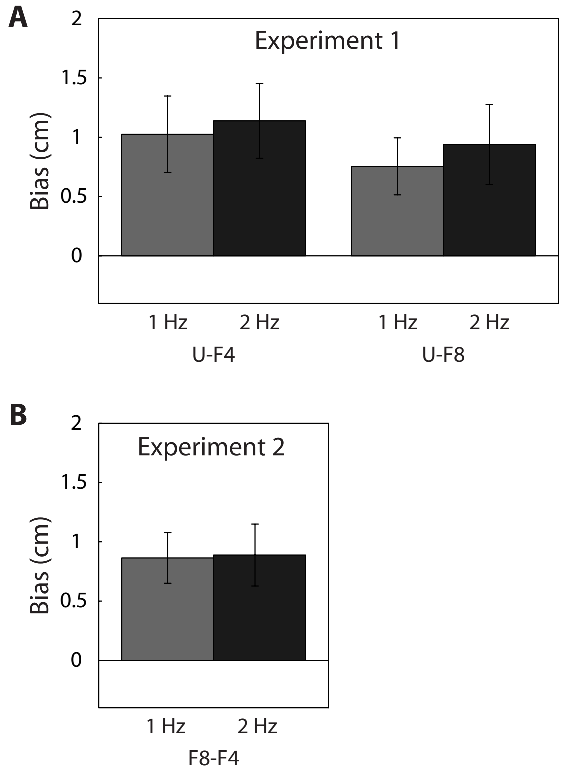

Figure 6.2. Average Points of Subjective Equality. Error bars indicate standard errors of the mean. (A) Conditions in Experiment 1: Unfilled – Filled 4 mm and Unfilled – Filled 8 mm at metronome frequencies of 1 and 2 Hz. (B) Conditions in Experiment 2: Filled 4 mm – Filled 8 mm at metronome frequencies of 1 and 2 Hz.

There were no effects on the thresholds of wavelength or movement speed. A repeated-measures analysis showed that the main effects of wavelength and movement speed and their interaction were non-significant ($F_{1,6}$ = 2.05, p = .202; $F_{1,6}$ = 2.78, p = .146; and $F_{1,6}$ = 0.44, p = .531, respectively). The overall average threshold was 2.13 cm.

Experiment 2

Figure 2B shows average PSEs for the two conditions in Experiment 2. Biases were significantly different from zero in both cases, with t_6 = 4.06, p = .003 and t_6 = 3.40, p = .007 for conditions 1 Hz and 2 Hz, respectively (one-tailed p-values). Positive biases indicate that a grating with a wavelength of 8 mm had to be longer than a grating with a wavelength of 4 mm in order for the two to be perceived as equally long. Lengths of 4-mm gratings are thus overestimated compared to 8-mm gratings. A paired-samples t test showed that there was no significant effect of movement speed (t_6 = 0.10, p = .928). The average bias in Experiment 2 was 0.88 cm, which corresponds to an effect size of 7.6%.

As in Experiment 1, there was no effect of movement speed on thresholds, as shown by a paired-samples t test (t_6 = 1.02, p = .349). The average threshold in Experiment 2 was 1.77 cm. A paired-samples t test on average thresholds per observer in Experiments 1 and 2 showed that thresholds in Experiment 2 were significantly smaller ($t_6 = 3.93$, $p = .008$).

Discussion

The results show that the lengths of extents filled with discrete filler elements are overestimated compared to less densely filled or unfilled extents. The strength of the illusion depends on the filler density. The amount of overestimation increases with the number of filler elements, as shown by the main effect of wavelength in Experiment 1 and the significant biases in Experiment 2. The results corroborate findings by Dresslar (1893). Importantly, these overestimations cannot be due to end point effects, and one might therefore interpret the illusion as a spatial distortion. Furthermore, we controlled for movement speed, so that speeds were the same for the two types of stimuli within a trial. As discussed in the Introduction, kinematic properties of hand movements play a determining role in haptic length perception. However, we found no evidence for an effect of doubling movement speed on the strength of the illusion in either experiment.

Although the intermediate stimulations at the fingertip are essentially non-informative about the length of the extent (the two end points suffice), we find that they affect perceived length. Moreover, the effects of filler density show that the intermediate stimulations affect perceived length parametrically, which raises the question as to the basis for the haptic filled-space illusion. Since the physical movement speed for the two stimuli within a trial was equal, the question is whether increased stimulation at the fingertip directly distorts the perception of kinematic variables of speed or time.

Since movement duration is a primary cue for perceived length, one could argue that filler elements affect perceived time. A number of studies have reported a filled-duration illusion, i.e., filled time intervals judged longer than empty ones (Buffardi, 1971; Craig, 1973; Goldstone and Goldfarb, 1963; Grimm, 1934; Otto Roelofs and Zeeman, 1951; Thomas and Brown, 1974). The illusion has been shown for the auditory modality (clicks and continuous tones), visual modality (flashes and continuous light), and tactile modality (vibrations applied to the index finger; short bursts or continuous vibrations). Thomas and Brown (1974) used different frequencies for end point tones and filler tones, and Buffardi (1971) showed that overestimations increase with the number of filler elements in all three modalities. Thus, if the haptic filled-space illusion for dynamic touch is essentially a variant of the filled-duration illusion, the influence of filler density in the former case must have a temporal character.

On the other hand, studies on tactile roughness perception show that discrimination performance is remarkably independent of scanning speed (Bensmaïa and Hollins, 2003; Lamb, 1983; Lederman, 1974; Lederman, 1983; Meftah et al., 2000; Smith et al., 2002; Vega-Bermudez et al., 1991). Thus, textures are generally perceived as equally rough, no matter how fast observers move their hands over the texture. This holds for both active and passive dynamic touch. More closely related to the present study are the experiments on frequency discrimination of gratings by Nefs et al. (2002), who remarked “there was considerable variation [in scanning speed] between and within trials" (p. 979). The studies on roughness perception and frequency discrimination of gratings suggest a different mechanism. The filler elements may affect perceived spatial extent, or the length signal after movement speed has been accounted for.

Another explanation for the haptic filled-space illusion could be that movement speed is perceived differently for different gratings. Two subsystems contribute to the sensation of speed of limb movement: proprioceptive stimulation in muscles and joints, and cutaneous stimulation at the contact between finger and stimulus. Interestingly, a recent study by Dépeault et al. (2008) investigated the ability to scale slip velocities through the use of a cylindrical drum rotating underneath the stationary fingertip. The surface of the drum could be smooth or covered with raised-dot patterns. For regular textures, the surface having a spatial period of 8 mm was perceived as moving slower than surfaces having a smaller spatial period (2-3 mm). In addition, smooth surfaces were perceived as moving slower than the textured surfaces. These effects are in agreement with the findings of the present paper. For example, consider a 4-mm and 8-mm grating of the same length. Movement duration is then the same, but perceived speed for the latter grating is lower. The length of the 4-mm grating would thus be overestimated, which is exactly what we found. However, the question is whether the effects on slip speed reported by Dépeault et al. (2008) are large enough to account for the present findings.

We found no effect of wavelength and movement speed on thresholds. Discrimination thresholds are a measure of the precision or reliability of observers' judgments. Apparently, the rubber pads that indicated the end points of the stimulus lengths were equally well discernable in all conditions. The finding of significantly lower thresholds in Experiment 2 is surprising, as this effect cannot be due to learning effects. Apparently, the precision with which grated lengths can be discriminated from each other is higher than when a grated length is compared with a raised block.

In conclusion, this study provides one of the first systematic investigations of parameters affecting the haptic filled-space illusion for dynamic touch. In contrast with previous studies, we have controlled for end point effects and within-trial differences in movement speed. We have shown that non-informative intermediate stimulations at the fingertip affect perceived length. The strength of the effect increases with the amount of intermediate stimulation. Since we know from previous research that length perception is primarily based on a kinematic mechanism, the increased stimulations possibly distort perception of kinematic variables of speed and time. Thus, the haptic filled-space illusion offers another opportunity to study the general principles underlying length estimation through dynamic touch.

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